Subject Code: 24BM11RC01 **R-24** **Reg No:** | | | | | | | | | | | |

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN
(Autonomous)
(Affiliated to Andhra University, Visakhapatnam)
I B.Tech. - I Semester Regular/ Supplementary Examinations, Jan – 2026

CALCULUS AND DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, CSE, IT, CSE-AIML, CSE-Cyber Security)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

UNIT-I

1. a. If $U = log(x^2 + y^2) + tan^{-1}\left(\frac{y}{x}\right)$ then show that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$
   b. If $U = x^2 + y^2 + z^2$ where $x = e^t$, $y = e^t \sin t$ and $z = e^t \cos t$ then find $\frac{dU}{dt}$

OR

2. a. Examine whether following functions are functionally dependent. If so, find the relation between them, where $u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$
   b. Find the Taylor's series expansion of $e^x \cos y$ about $x = 1, y = \pi/4$

UNIT-II

3. a. Find the maximum and minimum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$
   b. A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box such the material required for its construction is minimum.

OR

4. a. Using Lagrange's method of multipliers, find the point on the plane $ax + by + cz = p$ at which the function $f(x, y, z) = x^2 + y^2 + z^2$ has a minimum value and find this minimum value of $f(x, y, z)$
   b. Find the extreme values of $\sin x + \sin y + \sin(x + y)$

UNIT-III

5. a. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz dx dy$
   b. Change the order of integration and evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy \, dy dx$

OR

6. a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar co-ordinates
   b. Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

UNIT-IV

7. a. Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$
   b. A hot coal (temperature 120^0C) is immersed in ice water (temperature 0^0C). After 20 seconds the temperature of the coal drops to 80^0C. Assume that the ice water is kept at 0^0C. When does the temperature of the coal will reach to 20^0C.

OR

8. a. Solve the differential equation $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$
   b. Using the method of variation of parameters, solve $(D^2 + 4)y = \sec 2x$

UNIT-V

9. a. Find the Laplace Transform of $f(t) = \frac{e^{-at} - e^{-bt}}{t}$

   b. Find inverse Laplace transform of $F(s) = tan^{-1}\left(\frac{s}{2}\right)$

OR

10. a. Using Laplace Transforms, solve the initial value problem $\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0,\ given\ y(0) = 1, y'(0) = y''(0) = 2$

    b. Find the Laplace Transform of $f(t) = e^{-2t}sin3tcos2t$