Subject Code: 24BM11RC01 R-24 Reg No: | | | | | | | | | | | |

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(Autonomous)

(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. - I Semester Regular/ Supplementary Examinations, Jan – 2026

CALCULUS AND DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, CSE, IT, CSE-AIML, CSE-Cyber Security)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

UNIT-I

1. a. If $U = log(x^2 + y^2) + tan^{-1}\left(\frac{y}{x}\right)$ then show that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$

   b. If U = $x^2 + y^2 + z^2$ where x = e^t, y = e^t sint and z = e^t cost then find $\frac{dU}{dt}$

OR

2. a. Examine whether following functions are functionally dependent. If so, find the relation between them, where $u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$

   b. Find the Taylor's series expansion of e^xcosy about x = 1, y = $\pi/4$

UNIT-II

3. a. Find the maximum and minimum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

   b. A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box such the material required for its construction is minimum.

OR

4. a. Using Lagrange's method of multipliers, find the point on the plane ax + by + cz = p at which the function f(x, y, z) = $x^2 + y^2 + z^2$ has a minimum value and find this minimum value of f(x, y, z)

   b. Find the extreme values of sinx + siny + sin(x + y)

UNIT-III

5. a. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dzdxdy$

   b. Change the order of integration and evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy\, dydx$

OR

6. a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing into polar co-ordinates

   b. Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

UNIT-IV

7. a. Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

   b. A hot coal (temperature 120^0C) is immersed in ice water (temperature 0^0C). After 20 seconds the temperature of the coal drops to 80^0C. Assume that the ice water is kept at 0^0C. When does the temperature of the coal will reach to 20^0C.

OR

8. a. Solve the differential equation $(D^3 - 3D^2 + 4D - 2)y = e^x + cosx$

   b. Using the method of variation of parameters, solve $(D^2 + 4)y = sec2x$

UNIT-V

9. a. Find the Laplace Transform of $f(t)=\frac{e^{-at}-e^{-bt}}{t}$

b. Find inverse Laplace transform of $F(s) = tan^{-1}\left(\frac{s}{2}\right)$

OR

10. a. Using Laplace Transforms, solve the initial value problem $\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=0,\ given\ y(0)=1, y'(0)=y''(0)=2$

b. Find the Laplace Transform of $f(t) = e^{-2t} sin3t cos2t$

I B.Tech I sem Regular/supply Exams – Jan – 2026

Calculus and Differential Equations – Key

1.a. If $u = \log(x^2+y^2) + \tan^{-1}\left(\frac{y}{x}\right)$ then S.T $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2} \times 2x + \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{-y}{x^2}$$

$$= \frac{2x}{x^2+y^2} - \frac{y}{x^2+y^2} = \frac{2x-y}{x^2+y^2} \quad \text{—— (2M)}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2)(2) - (2x-y)(2x)}{(x^2+y^2)^2} = \cancel{\frac{2[x^2+y^2-2x^2+xy]}{(x^2+y^2)^2}}$$

$$= \frac{2[x^2+y^2-2x^2+xy]}{(x^2+y^2)^2} = \frac{2(y^2+xy-x^2)}{(x^2+y^2)^2} \quad \text{—— (1m)}$$

$$\frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2} + \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x}$$

$$= \frac{2y}{x^2+y^2} + \frac{x}{x^2+y^2} = \frac{2y+x}{x^2+y^2} \quad \text{—— (2m)}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{(x^2+y^2)(2) - (2y+x)(2y)}{(x^2+y^2)^2} = \frac{2[x^2+y^2-2y^2-xy]}{(x^2+y^2)^2}$$

$$= \frac{2[x^2-y^2-xy]}{(x^2+y^2)^2} \quad \text{—— (1m)}$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2[y^2+xy-x^2+x^2-y^2-xy]}{(x^2+y^2)^2} = 0$$

—— (1m)

Z

1.b. If $u = x^2+y^2+z^2$ where $x=e^t$, $y=e^t\sin t$ & $z=e^t\cos t$ then find $\frac{du}{dt}$.

Sol

$u = x^2+y^2+z^2$; $x=e^t$, $y=e^t\sin t$, $z=e^t\cos t$

$\frac{\partial u}{\partial x} = 2x$, $\frac{\partial u}{\partial y} = 2y$, $\frac{\partial u}{\partial z} = 2z$. — (1m)

$\frac{dx}{dt} = e^t$, $\frac{dy}{dt} = e^t\sin t + e^t\cos t$, $\frac{dz}{dt} = e^t\cos t - e^t\sin t$ — (2m)

$u = f(x,y,z)$, $x=\phi(t)$, $y=\psi(t)$, $z=\chi(t)$

then $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$ — (1m)

$\therefore \frac{du}{dt} = 2x \times e^t + 2y[e^t\sin t + e^t\cos t]$

$+ 2z[e^t\cos t - e^t\sin t]$ — (1m)

$= 2e^te^t + 2e^t\sin t \times e^t(\sin t + \cos t)$

$+ 2e^t\cos t(\cos t - \sin t)$

$= 2e^{2t}[1 + \sin^2 t + \sin t\cancel{\cos t} + \cos^2 t - \cos t\cancel{\sin t}]$

$= 2e^{2t}[1+1] = 4e^{2t}$. — (1m)

(or)

Given $u = x^2+y^2+z^2$, $x=e^t$, $y=e^t\sin t$, $z=e^t\cos t$

$\therefore u = e^{2t} + e^{2t}[\sin^2 t + \cos^2 t] = e^{2t}+e^{2t} = 2e^{2t}$ — (2m)

↳ (3m)

$$\therefore \frac{du}{dt} = 4e^{2t} \quad \text{—(2m)}$$

2.a. $u = \dfrac{x+y}{1-xy}, \quad v = \tan^{-1}x + \tan^{-1}y$

$$u_x = \frac{(1-xy)(1) - (x+y)(-y)}{(1-xy)^2} = \frac{1 - \cancel{xy} + \cancel{xy} + y^2}{(1-xy)^2}$$

$$= \frac{1+y^2}{(1-xy)^2} \quad \text{—(1m)}$$

$$\text{|||}^{ly} \quad u_y = \frac{1+x^2}{(1-xy)^2} \quad \text{—(1m)}$$

$$v_x = \frac{1}{1+x^2}, \quad v_y = \frac{1}{1+y^2} \quad \text{— (2m)}$$

WKT $J\left(\dfrac{u, v}{x, y}\right) = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} \dfrac{1+y^2}{(1-xy)^2} & \dfrac{1+x^2}{(1-xy)^2} \\ \dfrac{1}{1+x^2} & \dfrac{1}{1+y^2} \end{vmatrix}$

$\hookrightarrow$ (1m)

$$= \frac{1}{(1-xy)^2} - \frac{1}{(1-xy)^2} = 0 \quad \text{—(1m)}$$

$\therefore$ u, v are functionally dependent.

& the relation is $v = \tan^{-1}u$. — (1m)

Q.b. Let $f(x,y) = e^x \cos y$. $\quad f(1,\frac{\pi}{4}) = \frac{e}{\sqrt{2}}$ (4)

$$\left.\begin{aligned}
f_x &= e^x \cos y \\
f_y &= -e^x \sin y \\
f_{xx} &= e^x \cos y \\
f_{xy} &= -e^x \sin y \\
f_{yy} &= -e^x \cos y \\
f_{xxx} &= e^x \cos y \\
f_{xxy} &= -e^x \sin y \\
f_{xyy} &= -e^x \cos y \\
f_{yyy} &= e^x \sin y
\end{aligned}\right\} (3m)
\qquad
\left.\begin{aligned}
f_x(1,\tfrac{\pi}{4}) &= \tfrac{e}{\sqrt{2}} \\
f_y(1,\tfrac{\pi}{4}) &= -\tfrac{e}{\sqrt{2}} \\
f_{xx}(1,\tfrac{\pi}{4}) &= \tfrac{e}{\sqrt{2}} \\
f_{xy}(1,\tfrac{\pi}{4}) &= -\tfrac{e}{\sqrt{2}} \\
f_{yy}(1,\tfrac{\pi}{4}) &= -\tfrac{e}{\sqrt{2}} \\
f_{xxx}(1,\tfrac{\pi}{4}) &= \tfrac{e}{\sqrt{2}} \\
f_{xxy}(1,\tfrac{\pi}{4}) &= -\tfrac{e}{\sqrt{2}} \\
f_{xyy}(1,\tfrac{\pi}{4}) &= -\tfrac{e}{\sqrt{2}} \\
f_{yyy}(1,\tfrac{\pi}{4}) &= \tfrac{e}{\sqrt{2}}
\end{aligned}\right\} (1m)$$

By Taylor's series Expansion about the pt (a,b)

$$f(x,y) = f(a,b) + (x-a) f_x(a,b) + (y-b) f_y(a,b)$$
$$+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b) f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$
$$+ \frac{1}{3!}\left[(x-a)^3 f_{xxx}(a,b) + 3(x-a)^2(y-b) f_{xxy}(a,b) + 3(x-a)(y-b)^2 f_{xyy}(a,b)\right.$$
$$\left. + (y-b)^3 f_{yyy}(a,b)\right] + \cdots$$

$\longrightarrow$ (2m)

5

$$\therefore e^x \cos y = \frac{e}{\sqrt2} + (x-1)\frac{e}{\sqrt2} + \left(y-\frac{\pi}{4}\right)\left(\frac{-e}{\sqrt2}\right) + \frac{1}{2}\Big[(x-1)^2 \frac{e}{\sqrt2}$$

$$+ 2(x-1)\left(y-\frac{\pi}{4}\right)\left(\frac{-e}{\sqrt2}\right) + \left(y-\frac{\pi}{4}\right)^2\left(\frac{-e}{\sqrt2}\right)\Big]$$

$$+ \frac{1}{3!}\Big[(x-1)^3\frac{e}{\sqrt2} + 3(x-1)^2\left(y-\frac{\pi}{4}\right)\left(\frac{-e}{\sqrt2}\right) + 3(x-1)\left(y-\frac{\pi}{4}\right)^2\left(\frac{-e}{\sqrt2}\right)$$

$$+ \left(y-\frac{\pi}{4}\right)^3\left(\frac{e}{\sqrt2}\right)\Big] + \cdots$$

ie., $$e^x\cos y = \frac{e}{\sqrt2}\Big[1 + (x-1) - \left(y-\frac{\pi}{4}\right) + \frac{1}{2}(x-1)^2 - (x-1)\left(y-\frac{\pi}{4}\right)$$

$$- \frac{1}{2}\left(y-\frac{\pi}{4}\right)^2 + \frac{(x-1)^3}{6} - \frac{(x-1)^2\left(y-\frac{\pi}{4}\right)}{2}$$

$$- \frac{(x-1)\left(y-\frac{\pi}{4}\right)^2}{2} + \frac{\left(y-\frac{\pi}{4}\right)^3}{6}\Big] + \cdots$$

— (m)

3(a). Let $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

$$\left.\begin{aligned} f_x &= 3x^2 + 3y^2 - 30x + 72 \\ f_y &= 6xy - 30y \end{aligned}\right\} \text{ — (1m)}$$

$$\begin{aligned} f_x = 0 \\ f_y = 0 \end{aligned} \Rightarrow \begin{aligned} &3(x^2+y^2-10x+24) = 0 \longrightarrow (1) \\ &6y(x-5) = 0 \longrightarrow (2) \end{aligned}$$

$\Rightarrow y = 0$ or $x = 5$

when $x = 5$, eq(1) becomes $25 + y^2 - 50 + 24 = 0$

$\Rightarrow y^2 - 1 = 0 \Rightarrow y = \pm 1$

when $y=0$, $x^2-10x+24=0$

$(x-4)(x-6)=0$

$\Rightarrow x=4,6$

$\therefore$ The stationary pts are $(5,1)(5,-1)(4,0)(6,0)$ — (1m)

Now $r=f_{xx}=6x-30$, $s=f_{xy}=6y$, $t=f_{yy}=6x-30$ — (1m)

At $(5,1)$, $(5,-1)$

$r=0$, $s=\pm 6$, $t=0$

$\therefore rt-s^2=-36<0$.

so $(5,1)$ $(5,-1)$ are saddle points ie., f attains neither its maximum nor minimum at $(5,1)(5,-1)$ — (1m)

At $(4,0)$, $r=-6$, $s=0$, $t=-6$

$\therefore rt-s^2=36>0$ & $r=-6<0$.

so f attains its maximum at $(4,0)$ — (1m)

At $(6,0)$, $r=6$, $s=0$, $t=6$

$\therefore rt-s^2=36>0$ & $r=6>0$

so f attains its minimum at $(6,0)$ — (1m)

The maximum value is $f(4,0)=112$ }
minimum value is $f(6,0)=108$ } (1m)

3(b) Let x, y, z be the dimensions of the box open at the top. (length, breadth, height) 9

$\therefore$ The volume of the box is $V = xyz = 32 \longrightarrow (1)$

surface area. $S = xy + 2yz + 2zx$. } — (1m)

It is to minimize the surface area subject to the condition (1)

$\therefore$ The Lagrangean function is

$f = S + \lambda V$ ie., $F = xy + 2yz + 2zx + \lambda(xyz - 32)$ — (1m)

$\frac{\partial f}{\partial x} = 0 \Rightarrow y + 2z + \lambda(yz) = 0 \longrightarrow (2)$

$\frac{\partial f}{\partial y} = 0 \Rightarrow x + 2z + \lambda(xz) = 0 \longrightarrow (3)$ (2m)

$\frac{\partial f}{\partial z} = 0 \Rightarrow 2y + 2x + \lambda(xy) = 0 \longrightarrow (4)$

from Eq(2), (3) & (4)

$-\lambda = +\frac{(y+2z)}{yz} = +\frac{(x+2z)}{xz} = \frac{+2(y+x)}{xy}$ — (1m)

$\frac{y+2z}{yz} = \frac{x+2z}{xz}$

$\Rightarrow xy + 2xz = xy + 2yz$

$\Rightarrow \boxed{x = y}$

$\frac{y+2z}{yz} = \frac{2(y+x)}{xy}$

$\Rightarrow xy + 2xz = 2yz + 2xz$

$\Rightarrow \boxed{x = 2z}$

$\therefore \boxed{x = y = 2z}$ — (1m)

Subf. in Eq (1) we get

$$(x)(x)\left(\frac{x}{2}\right) = 32$$

$$\Rightarrow x^3 = 64 \Rightarrow x = 4.$$

$\therefore x = 4ft,\ y = 4ft\ z = 2ft.$ are the required dimension. —(1m)

4(a). Let $\phi = ax + by + cz = p.$ —→ ①

$$f(x,y,z) = x^2 + y^2 + z^2.$$

we find the minimum of f subject to the condition ①

$\therefore$ The Lagrangean function is

$$F = f + \lambda\phi$$

$$F = x^2 + y^2 + z^2 + \lambda(ax + by + cz - p) \quad \text{—(1m)}$$

$$\frac{\partial F}{\partial x} = 0 \Rightarrow 2x + a\lambda = 0 \longrightarrow (2)$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow 2y + b\lambda = 0 \longrightarrow (3)$$

$$\frac{\partial F}{\partial z} = 0 \Rightarrow 2z + c\lambda = 0 \longrightarrow (4)$$

(2m)

from Eq (2) (3) & (4)

$$-\frac{\lambda}{2} = \frac{x}{a} = \frac{y}{b} = \frac{z}{c}. \quad \text{—(1m)}$$

$\Rightarrow \quad y = \frac{bx}{a}, \quad z = \frac{cx}{a}.$

Sub. in Eq (1) we get.

$$ax + \frac{b^2x}{a} + \frac{c^2x}{a} = p.$$

$$\Rightarrow (a^2+b^2+c^2)x = p \Rightarrow x = \frac{ap}{a^2+b^2+c^2}.$$

$$y = \frac{bp}{a^2+b^2+c^2}, \quad z = \frac{cp}{a^2+b^2+c^2} \quad \text{—— (2m)}$$

$\therefore$ ~~the stationary~~ The required point is

$$\left(\frac{ap}{a^2+b^2+c^2}, \frac{bp}{a^2+b^2+c^2}, \frac{cp}{a^2+b^2+c^2}\right)$$

& the minimum value is $\frac{a^2p^2+b^2p^2+c^2p^2}{(a^2+b^2+c^2)^2} = \frac{p^2}{a^2+b^2+c^2}.$ —(1m)

4.(b). Let $u(x,y) = \sin x + \sin y + \sin(x+y)$

$u_x = \cos x + \cos(x+y), \quad u_y = \cos y + \cos(x+y)$ —(1m)

$u_x = 0, u_y = 0 \Rightarrow \cos x + \cos(x+y) = 0 \longrightarrow (1)$

$\cos y + \cos(x+y) = 0 \longrightarrow (2)$

from Eq (1) & (2)

$\cos x = \cos y. \Rightarrow \boxed{x = y}.$ —— (1m)

$\therefore$ Eq (1) becomes $\cos x + \cos 2x = 0$

$\Rightarrow 2\cos\frac{3x}{2}\cos\frac{x}{2} = 0$

$\cos\frac{x}{2} = 0$, $\cos\frac{3x}{2} = 0$

$\Rightarrow \frac{x}{2} = \pm\frac{\pi}{2}$, $\frac{3x}{2} = \pm\frac{\pi}{2}$

$\Rightarrow x = \pm\pi$, $x = \pm\frac{\pi}{3}$

$\therefore$ The Stationary points are.

$\left(\pm\frac{\pi}{3}, \pm\frac{\pi}{3}\right)$ $(\pm\pi, \pm\pi)$ —— (1m)

Now $r = u_{xx} = -\sin x - \sin(x+y)$

$s = u_{xy} = -\sin(x+y)$

$t = u_{yy} = -\sin y - \sin(x+y)$ (2M)

At $(\pm\pi, \pm\pi)$, $r = 0$, $s = 0$, $t = 0$

$rt - s^2 = 0$ —

So it needs further investigation.

At $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ $r = -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = -\sqrt{3}$.

$s = -\frac{\sqrt{3}}{2}$, $t = -\sqrt{3}$

$\therefore rt - s^2 = 3 - \left(\frac{-\sqrt{3}}{2}\right)^2 = 3 - \frac{3}{4} = \frac{9}{4} > 0$

& $r = -\sqrt{3} < 0$. So f attains maximum at $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

— (1m)

At $\left(-\frac{\pi}{3}, -\frac{\pi}{3}\right)$; $r = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3} > 0$

$s = \frac{\sqrt{3}}{2}$, $t = \sqrt{3}$.

$\therefore \quad rt - s^2 = 3 - \frac{3}{4} = \frac{9}{4} > 0 \;\&\; r > 0$ (14)

so f attains minimum at $\left(\frac{-\pi}{3}, \frac{-\pi}{3}\right)$.

The maximum value is $\frac{3\sqrt{3}}{2}$ & the minimum value is $-\frac{3\sqrt{3}}{2}$ (1M)

5.a. $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx dy$.

$= \int_0^1 \int_{y^2}^1 x(z)_0^{1-x} \, dx dy$ —— (1M)

$= \int_0^1 \int_{y^2}^1 x(1-x) \, dx dy$ —— (1M)

$= \int_0^1 \int_{y^2}^1 (x - x^2) \, dx dy = \int_0^1 \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{y^2}^1 dy$ —(1M)

$= \int_0^1 \left[\frac{1}{2}(1-y^4) - \frac{1}{3}(1-y^6)\right] dy$

$\frac{1}{2} - \frac{1}{3} = \frac{3-2}{6}$

$= \int_0^1 \left(-\frac{y^4}{2} + \frac{1}{6} + \frac{y^6}{3}\right) dy$ —(2M)

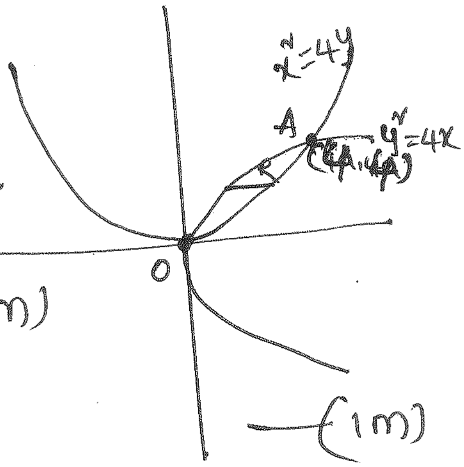
$= \left[\frac{y}{6} - \frac{1}{2}\frac{y^5}{5} + \frac{1}{3}\frac{y^7}{7}\right]_0^1 = \frac{1}{6} - \frac{1}{10} + \frac{1}{21}$

$= \frac{35 - 21 + 10}{210} = \frac{24}{210}$

$= \frac{4}{35}$. —(2m)

5.b. Given $y = \frac{x^2}{4}$, $y = 2\sqrt{x}$

i.e., $x^2 = 4y$; $y^2 = 4x$.



over R ; x varies from

$\frac{y^2}{4}$ to $2\sqrt{y}$ } (2m)

y varies from 0 to 4

— (1m)

$$\therefore \int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy \, dy \, dx = \int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} xy \, dx \, dy \quad \text{— (1m)}$$

$$= \int_0^4 y \left[\frac{x^2}{2}\right]_{y^2/4}^{2\sqrt{y}} dy$$

$$= \frac{1}{2}\int_0^4 y\left[4y - \frac{y^4}{16}\right] dy \quad \text{— (1m)}$$

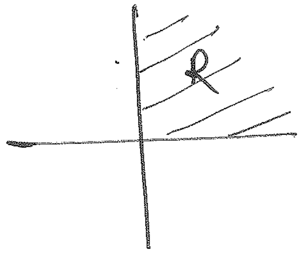
$$= \frac{1}{2}\int_0^4 \left(4y^2 - \frac{y^5}{16}\right) dy$$

$$= \frac{1}{2}\left\{4\frac{y^3}{3} - \frac{1}{16}\frac{y^6}{6}\right\}_0^4$$

$$= \frac{1}{2}\left\{\frac{4}{3}\times 4^3 - \frac{1}{4^2}\,\frac{4^6}{6}\right\}$$

$$= \frac{4^4}{2}\left[\frac{1}{3}-\frac{1}{6}\right] = \frac{\overset{8}{\cancel{16}}\times\overset{8}{\cancel{16}}}{\cancel{2}}\times\frac{1}{\cancel{6}_3} = \frac{64}{3}$$ —(2m)

6a. $\displaystyle\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$



put $x = r\cos\theta,\ y = r\sin\theta$.

$\therefore x^2+y^2 = r^2$ & $J\left(\frac{x,y}{r,\theta}\right) = r$

so replace $dx\,dy$ by $r\,dr\,d\theta$. —(2m)

over the given region 'R' : r varies from 0 to ∞

θ varies from 0 to $\frac{\pi}{2}$

—(2m)

$$\therefore \int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy = \int_0^{\pi/2}\int_0^\infty e^{-r^2}\,r\,dr\,d\theta$$ —(1m)

$$= \int_0^{\pi/2} d\theta \times -\frac{1}{2}\int_0^\infty -2r\,e^{-r^2}\,dr$$

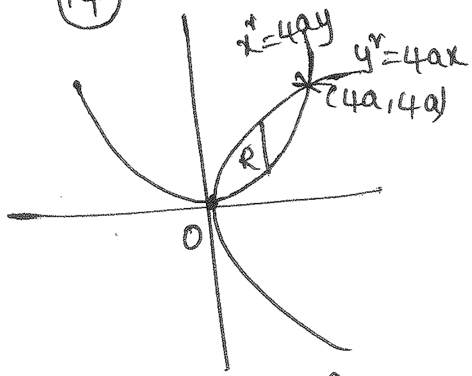
$$= (\theta)_0^{\pi/2}\times -\frac{1}{2}\left[e^{-r^2}\right]_0^\infty = \frac{\pi}{2}\times -\frac{1}{2}[0-1]$$

$$= \frac{\pi}{4}$$ // (2m

(14)

6.b

Given $y^2 = 4ax$ & $x^2 = 4ay$



— (1m)

~~overR~~

Area bounded in xy-plane

$$= \iint_R dxdy \quad \text{— (1m)}$$

over R : ~~x~~ y varies from $\frac{x^2}{4a}$ to $2\sqrt{ax}$

x varies from 0 to 4a. — (1m)

$$\therefore \text{The required area} = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx.$$

$$= \int_0^{4a} (y)_{x^2/4a}^{2\sqrt{ax}} dx \quad \text{— (1m)}$$

$$= \int_0^{4a} \left(2a^{1/2} x^{1/2} - \frac{x^2}{4a}\right) dx. \quad \text{— (1m)}$$

$$= \left[2a^{1/2} \frac{x^{3/2}}{3/2} - \frac{1}{4a}\frac{x^3}{3}\right]_0^{4a}$$

$$= \frac{4}{3} a^{1/2} (4a)^{3/2} - \frac{1}{\cancel{4a}} \times \frac{4^{\cancel{3}\,2} a^{\cancel{3}\,2}}{3}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3}. \quad \text{— (2m)}$$

7.a. $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$ —— (1) (15)

Given Eqn. is in the form $M\,dx + N\,dy = 0$

where $M = 3x^2y^4 + 2xy$, $N = 2x^3y^3 - x^2$

$$\frac{\partial M}{\partial y} = 12x^2y^3 + 2x, \quad \frac{\partial N}{\partial x} = 6x^2y^3 - 2x$$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ so Eq (1) is not Exact —— (1m)

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y(3x^2y^3+2x)}\left[6x^2y^3 - 2x - 12x^2y^3 - 2x\right]$$

$$= \frac{-6x^2y^3 - 4x}{y(3x^2y^3+2x)} = -\frac{2}{y} = g(y)$$

$$\therefore I.F = e^{\int g(y)\,dy} = e^{\int -\frac{2}{y}\,dy} = e^{-2\ln y} = \frac{1}{y^2}$$

multiplying Eq (1) by $\frac{1}{y^2}$ on bs we get —— (2m)

$$\left(3x^2y^2 + \frac{2x}{y}\right)dx + \left(2x^3y - \frac{x^2}{y^2}\right)dy = 0 \text{ —— (2)}$$

Clearly Eq (2) is in the form $M_1\,dx + N_1\,dy = 0$

which is an Exact D.E —— (1m)

$$\left[\text{As } \frac{\partial M_1}{\partial y} = 6x^2y - \frac{2x}{y^2} = \frac{\partial N_1}{\partial x}\right]$$

$\therefore$ The G.S is $\int\limits_{y\,\text{const}} M_1\,dx + \int \left(\text{terms of } N_1 \text{ not containing } x\right) dy = C$

—— (1m)

$$\Rightarrow \int_{y\ \text{const}} \left(3x^2y^2 + \frac{2x}{y}\right) dx + \int 0 = c$$

$$\Rightarrow x^3y^2 + \frac{x^2}{y} = c.$$ is the required solution — (2m)

7.b. By Newton's Law of cooling

Rate of change in temperature of a body is proportional to difference in templ. of the body and that of surrounding medium. — (1m)

Let θ be the templ. of a body at any time 't', and let θ_0 be the templ. of the surrounding medium.

$$\therefore \frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\Rightarrow \frac{d\theta}{dt} = -K(\theta - \theta_0) \longrightarrow (1)$$ Where K is a +ve constant

By solving (1) we get $\boxed{\theta = \theta_0 + ce^{-Kt}} \rightarrow (2)$ — (1m)

c is a constant.

Given $\theta_0 = 0$, $\theta = 120^\circ C$ when $t = 0$.

$$\therefore 120 = 0 + ce^{-K \times 0} \Rightarrow \boxed{c = 120} \quad \text{— (1m)}$$

Given $\theta = 80^\circ C$ when $t = 20$ sec.

$\therefore\ 80 = 0 + 120 e^{-k\times 20}$

$\Rightarrow \quad \frac{2}{3} = e^{-20k} \Rightarrow \boxed{k = -\frac{1}{20}\ln\left(\frac{2}{3}\right)}$ — (2m)

$\therefore\ \theta = 0 + 120 e^{\frac{1}{20}\ln\left(\frac{2}{3}\right)t}$

when $\theta = 20°C$.

$$2\cancel{0} = 12\cancel{0}\ e^{\frac{1}{20}\ln\left(\frac{2}{3}\right)t}$$

$$\Rightarrow \quad \frac{1}{6} = e^{\frac{1}{20}\ln\left(\frac{2}{3}\right)t}$$

$$\Rightarrow \quad \ln\left(\frac{1}{6}\right) = \frac{1}{20}\ln\left(\frac{2}{3}\right)t \quad \text{— (1m)}$$

$$\therefore t = 20\left[\frac{\ln(1/6)}{\ln(2/3)}\right]$$

$$= 20\left[\frac{\ln 1 - \ln 6}{\ln 2 - \ln 3}\right] = 88.38 \text{ sec.}$$

— (1m)

The templ. will be $\underline{\underline{20°C}}$ after 88.38 secs.

8.a. $(D^3 - 3D^2 + 4D - 2)\,y = e^x + \cos x$.

Given eqn. is in the form $f(D)y = Q$

where $f(D) = D^3 - 3D^2 + 4D - 2$, $Q = e^x + \cos x$

$\therefore$ The A.E is $f(m) = 0$ i.e., $m^3 - 3m^2 + 4m - 2 = 0$

$$m = 1,\ 1 \pm i$$

$\therefore\ y_c = c_1 e^x + e^x[c_2 \cos x + c_3 \sin x]$ — (2m) where c_1, c_2, c_3 are arbitrary constants.

$$y_p = \frac{1}{f(D)} Q = \frac{1}{D^3 - 3D^2 + 4D - 2} e^x + \frac{1}{D^3 - 3D^2 + 4D - 2} \cos x$$

$$= I_1 + I_2$$

$$I_1 = \frac{1}{D^3 - 3D^2 + 4D - 2} e^x$$

WKT $\frac{1}{f(D)} e^{ax} = \frac{x}{f'(a)} e^{ax}$

if $f(a) = 0$

& $f'(a) \neq 0$

$$\therefore I_1 = \frac{x}{f'(1)} e^x$$

where $f(D) = D^3 - 3D^2 + 4D - 2$

$f'(D) = 3D^2 - 6D + 4$

$\therefore f'(1) = 3 - 6 + 4 = 1$

$$\therefore I_1 = \frac{xe^x}{1} = xe^x \text{ — (2m)}$$

$$I_2 = \frac{1}{D^3 - 3D^2 + 4D - 2} \cos x.$$

put $D^2 = -1$

$$\therefore I_2 = \frac{1}{-D + 3 + 4D - 2} \cos x$$

$$= \frac{1}{3D + 1} \cos x$$

$$= \frac{3D - 1}{9D^2 - 1} \cos x$$

Again put $D^2 = -1$

$$\therefore I_2 = \frac{3D - 1}{-9 - 1} \cos x$$

$$= \frac{-1}{10} [3x - \sin x - \cos x]$$

$$= \frac{1}{10} [3 \sin x + \cos x] \text{ — (2m)}$$

$$\therefore y_p = xe^x + \frac{1}{10} [3 \sin x + \cos x].$$

Hence the G.S is $y = y_c + y_p$ — (1m)

i.e., $y = c_1 e^x + e^x [c_2 \cos x + c_3 \sin x] + xe^x + \frac{1}{10} [3 \sin x + \cos x]$

8.b. Given eqn. is $f(D)y = R$

where $f(D) = D^2+4$, $R = \sec 2x$

$\therefore$ The A.E is $f(m)=0$ i.e., $m^2+4=0$

$\Rightarrow m = \pm 2i$

$\therefore y_c = c_1\cos 2x + c_2 \sin 2x$ — (2m)

$= c_1 u + c_2 v$

where $u = \cos 2x$, $v = \sin 2x$.

$\therefore u' = -2\sin 2x$, $v' = 2\cos 2x$

$\therefore W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - vu' = (\cos 2x)(2\cos 2x) - (\sin 2x)(-2\sin 2x)$

$= 2[\cos^2 2x + \sin^2 2x] = 2 \neq 0$ — (1m)

Take $y_p = Au + Bv = A\cos 2x + B\sin 2x$

where $A = -\int \frac{vR}{W}dx$, $B = \int \frac{uR}{W}dx$ — (1m)

$\therefore A = -\int \frac{\sin 2x \times \sec 2x}{2} dx$; $B = \int \frac{\cos 2x \times \sec 2x}{2} dx$

$= -\frac{1}{2}\int \tan 2x\, dx$ $\qquad = \frac{1}{2}\int 1\, dx$

$= +\frac{1}{2}\frac{\ln(\cos 2x)}{2} = \frac{\ln\cos 2x}{4}$ — (1m) $\qquad = \frac{x}{2}$ — (1m)

$$\therefore y_p = \frac{\cos 2x}{4} \ln\cos 2x + \frac{x}{2}\sin 2x.$$

Hence the G.S is $y = y_c + y_p$. — (1m)

9.a. $\dfrac{e^{-at} - e^{-bt}}{t}$

let $g(t) = e^{-at} - e^{-bt}$

$$\therefore L[g(t)] = L[e^{-at}] - L[e^{-bt}]$$

$$= \frac{1}{s+a} - \frac{1}{s+b} = G(s) \quad \text{— (2m)}$$

WKT $L\left[\dfrac{g(t)}{t}\right] = \displaystyle\int_s^\infty L[g(t)]\,ds$ (or) $\displaystyle\int_s^\infty G(s)\,ds$ — (2m)

$$\therefore L\left[\frac{e^{-at} - e^{-bt}}{t}\right] = \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds \quad \text{— (1m)}$$

$$= \left[\ln(s+a) - \ln(s+b)\right]_s^\infty \quad \text{— (1m)}$$

$$= \ln\left(\frac{s+a}{s+b}\right)\Big]_s^\infty = -\ln\left(\frac{s+a}{s+b}\right)$$

$$= \ln\left(\frac{s+b}{s+a}\right) \quad \text{— (1m)}$$

9.b. Let $F(s) = \tan^{-1}\left(\frac{s}{2}\right)$

$$\therefore \frac{d[F(s)]}{ds} = \frac{1}{1+\left(\frac{s}{2}\right)^2} \times \frac{1}{2} = \frac{4}{4+s^2} \times \frac{1}{2}$$

$$= \frac{2}{s^2+4}. \quad \text{--- (2m)}$$

$$\therefore L^{-1}\left[\frac{d[F(s)]}{ds}\right] = L^{-1}\left[\frac{2}{s^2+4}\right] = \sin 2t = f(t)$$

— (2m)

WKT If $L^{-1}[F(s)] = f(t)$ then $L^{-1}\left[\frac{d[F(s)]}{ds}\right] = -tf(t)$

ie., $L^{-1}[F(s)] = -\frac{1}{t} L^{-1}\left[\frac{d[F(s)]}{ds}\right]$ — (2m)

$$\therefore L^{-1}\left[\tan^{-1}\left(\frac{s}{2}\right)\right] = -\frac{\sin 2t}{t} \quad \text{--- (1m)}$$

10.a. Given eq. is $y''' + 2y'' - y' - 2y = 0$. → ①

$y(0) = 1$, $y'(0) = y''(0) = 2$

Applying L.T on bs of Eq (1) we get

$$L[y'''] + 2L[y''] - L[y'] - 2L[y] = 0$$

Let $L[y(t)] = Y(s)$

$$\therefore s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + 2[s^2 Y(s) - s y(0) - y'(0)]$$
$$- [sY(s) - y(0)] - 2Y(s) = 0 \quad \text{--- (1m)}$$

$$\Rightarrow (s^3+2s^2-s-2)Y(s) - s^2 - 2s - 2 - 2s - 4 + 1 = 0$$

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$$\Rightarrow (s^3+2s^2-s-2)Y(s) = s^2+4s+5$$

$$\Rightarrow \quad Y(s) = \frac{s^2+4s+5}{s^3+2s^2-s-2} \quad \text{—— (2m)}$$

Apply ILT on bs weget

$$L^{-1}[Y(s)] = L^{-1}\left[\frac{s^2+4s+5}{s^3+2s^2-s-2}\right]$$

$$\Rightarrow \quad y(t) = L^{-1}\left[\frac{s^2+4s+5}{s^3+2s^2-s-2}\right] \quad \text{—— (1m)}$$

WKT $\quad \dfrac{s^2+4s+5}{s^3+2s^2-s-2} = \dfrac{s^2+4s+5}{(s-1)(s+1)(s+2)}$

$$= \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$\Rightarrow s^2+4s+5 = A(s+1)(s+2) + B(s-1)(s+2) + C(s-1)(s+1)$$

put $s=1$, weget $\quad 10 = A(2)(3) \Rightarrow \boxed{A = 5/3}$

put $s=-1$, weget $\quad 2 = B(-2)(1) \Rightarrow \boxed{B=-1}$

put $s=-2$, weget $\quad 1 = C(-3)(-1) \Rightarrow \boxed{C = 1/3}$

$$\therefore \frac{s^2+4s+5}{s^3+2s^2-s-2} = \frac{5}{3(s-1)} - \frac{1}{s+1} + \frac{1}{3(s+2)} \quad \text{—— (2m)}$$

$$\therefore y(t) = \frac{5}{3}L^{-1}\left[\frac{1}{s-1}\right] - L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{3}L^{-1}\left[\frac{1}{s+2}\right]$$

$\therefore y(t) = \frac{5}{3}e^{t} - e^{-t} + \frac{1}{3}e^{-2t}$ is the required solution. — (1m)

Z.

10.b.

WKT $\sin 3t \cos 2t = \frac{1}{2}[\sin 5t + \sin t]$

$\therefore L[\sin 3t \cos 2t] = \frac{1}{2}\{L[\sin 5t] + L[\sin t]\}$

$= \frac{1}{2}\left\{\frac{5}{s^2+25} + \frac{1}{s^2+1}\right\} = F(s)$

— (2m)

By first shifting property

$L[e^{at} f(t)] = \{L[f(t)]\}_{s \to s-a}$ — (2m)

$\therefore L[e^{-2t} \sin 3t \cos 2t] = \{L[\sin 3t \cos 2t]\}_{s \to s+2}$

$= \frac{1}{2}\left[\frac{5}{s^2+25} + \frac{1}{s^2+1}\right]_{s \to s+2}$ — (1m)

$= \frac{1}{2}\left[\frac{5}{(s+2)^2+25} + \frac{1}{(s+2)^2+1}\right]$

$= \frac{1}{2}\left\{\frac{5}{s^2+4s+29} + \frac{1}{s^2+4s+5}\right\}$. — (2m)

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