Subject Code: 24BM11RC01 R-24

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(Autonomous)

(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. - I Semester Regular/ Supplementary Examinations, Jan – 2026

CALCULUS AND DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, CSE, IT, CSE-AIML, CSE-Cyber Security)

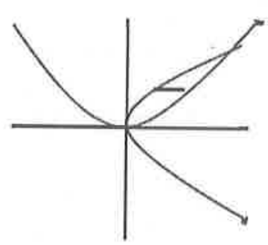
1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. Max Marks: 70

SCHEME OF VALUATION

Q No	Question
1(a)	**If $U = log(x^2 + y^2) + tan^{-1}\left(\frac{y}{x}\right)$ then show that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$** $\frac{\partial u}{\partial x} = \frac{2x-y}{x^2+y^2}$ --------------------------------→ 2M $\frac{\partial u}{\partial y} = \frac{2y+x}{x^2+y^2}$ --------------------------------→ 2M $\frac{\partial^2 u}{\partial x^2} = \frac{(2y^2+xy-x^2)}{(x^2+y^2)^2}$ --------------------------------→ 1M $\frac{\partial^2 u}{\partial y^2} = \frac{(2x^2-y^2-xy)}{(x^2+y^2)^2}$ --------------------------------→ 1M Therefore $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ --------------------------------→ 1M
1(b)	**If U = x^2+ y^2+z^2 where x = e^t, y = e^t sint and z = e^t cost then find $\frac{dU}{dt}$** $\frac{\partial U}{\partial x} = 2x, \frac{\partial U}{\partial y} = 2y, \frac{\partial U}{\partial z} = 2z$ --------------------------------→ 1M $\frac{dx}{dt} = e^t, \frac{dy}{dt} = e^t(\sin t + cost), \frac{dz}{dt} = e^t(\cos t - sint)$ --------------------------------→ 3M WKT $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$ --------------------------------→ 1M $\frac{du}{dt} = 4e^{2t}$ --------------------------------→ 2 M (or) Finding derivative using substitution is also valid U = $2e^{2t}$; $\frac{du}{dt} = 4e^{2t}$
2(a)	**Examine whether following functions are functionally dependent. If so, find the relation between them, where $u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$** $u_x = \frac{1+y^2}{(1-xy)^2}$; $u_y = \frac{1+x^2}{(1-xy)^2}$ --------------------------------→ 2M $v_x = \frac{1}{1+x^2}$; $v_y = \frac{1}{1+y^2}$ --------------------------------→ 2M $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = 0$ --------------------------------→ 2M $\therefore u, v$ are functionally dependent and the relation between them is $v = tan^{-1}u$ -----→ 1M

2(b)	**Find the Taylor's series expansion of $e^x\cos y$ about $x = 1, y = \pi/4$** Given $f = e^x cos y,\ f(1,\pi/4) = e/\sqrt{2}$ $f_x = e^x cos y;\quad f_x(1,\pi/4) = e/\sqrt{2}$ $f_y = -e^x \sin y;\quad f_y(1,\pi/4) = -e/\sqrt{2}$ $f_{xx} = e^x cos y;\quad f_{xx}(1,\pi/4) = e/\sqrt{2}$ $f_{xy} = -e^x \sin y;\quad f_{xy}(1,\pi/4) = -e/\sqrt{2}$ $f_{yy} = -e^x cos y;\ f_{yy}(1,\pi/4) = -e/\sqrt{2}$ ---------→ 4M Taylor's series expansion of $f(x,y)$ about the point $(a,\ b)$ ---------→ 2M $f(x,y) = \frac{e}{\sqrt{2}}\left[1 + (x-1) - \left(y-\frac{\pi}{4}\right) + \frac{1}{2}(x-1)^2 - (x-1)\left(y-\frac{\pi}{4}\right) - \frac{1}{2}\left(y-\frac{\pi}{4}\right)^2 + --\right]$ ---------→ 1M
3(a)	**Find the maximum and minimum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.** Let $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ $f_x = 3x^2 + 3y^2 - 30x + 72,\ f_y = 6xy - 30y$ ---------→ 1M $f_x = 0,\ f_y = 0 \Rightarrow\ y = 0\ or\ x = 5$ ---------→ 1M The stationary points are $(5,\ 1), (5,-1),\ (4,\ 0), (6,\ 0)$ ---------→ 1M $r = f_{xx} = 6x - 30,\ s = f_{xy} = 6y, t = f_{yy} = 6x - 30$ ---------→ 1M. At $(5,\ 1), (5,-1)\ rt - s^2 = -36 < 0$ so f attains neither its maximum nor its minimum at these points and these points are called saddle points. At $(4,\ 0), r = -6, s = 0, t = -6$ ---------→ 1M $rt - s^2 > 0\ and\ r < 0$ so f attains its maximum at (4, 0). At $(6,\ 0), r = 6, s = 0, t = 6$ ---------→ 1M $rt - s^2 > 0\ and\ r > 0$ so f attains its minimum at (6, 0). The maximum value is 112, minimum value is108 ---------→ 1M
3(b)	**A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box such the material required for its construction is minimum.** Let x, y, z (in ft) be the length, width and height of the rectangular box which is open at the top. The volume of the box is $V = xyz = 32$ and the surface area is $S = xy + 2yz + 2zx$. ---→ 1M $\therefore$ The Lagrangean function is $F = S + \lambda V = xy + 2yz + 2zx + \lambda(xyz - 32)$ --------→ 1M $f_x = 0, f_y = 0,\ f_z = 0$ $\Rightarrow y + 2z + \lambda yz = 0,\ x + 2z + \lambda xz = 0,\ 2y + 2x + \lambda xy = 0$ ---------→ 2M $\therefore\ \frac{(y+2z)}{yz} = \frac{(x+2z)}{xz} = \frac{2(y+x)}{xy} = -\lambda$ ---------→ 1M $\Rightarrow x = y = 2z$ ---------→ 1M $\therefore x = y = 4\ ft\ and\ z = 2\ ft$ ---------→ 1M
4(a)	**Using Lagrange's method of multipliers, find the point on the plane ax + by + cz = p at which the function $f(x, y, z) = x^2 + y^2 + z^2$ has a minimum value and find this minimum value of f(x, y, z).** Given $f = x^2 + y^2 + z^2$ and $\phi = ax + by + cz - p = 0$ The Lagrangean function is $F = f + \lambda\phi = x^2 + y^2 + z^2 + \lambda\,(ax + by + cz - p) = 0$ - 1M

	$F_x = 0, F_y = 0, F_z = 0 \Rightarrow 2x + a\lambda = 0, 2y + b\lambda = 0$ *and* $2z + c\lambda = 0$ ------------→ 2M $\therefore \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = -\frac{\lambda}{2}$ ------------→ 1M $\therefore x = \frac{ap}{a^2+b^2+c^2}, y = \frac{bp}{a^2+b^2+c^2}, z = \frac{cp}{a^2+b^2+c^2}$ ------------→ 2M The minimum value of the function is $\frac{p^2}{a^2+b^2+c^2}$ ------------→ 1M
4 (b)	**Find the extreme values of sinx + siny + sin(x + y)** Let $f(x,y) = sinx + siny + \sin(x+y)$ $f_x = \cos x + \cos(x+y)$, $f_y = cosy + \cos(x+y)$ --------------------→ 1M $f_x = 0, \ f_y = 0 \Rightarrow \ x = y$ --------------------→ 1M The stationary points are $\left(\pm\frac{\pi}{3}, \pm\frac{\pi}{3}\right), (\pm\pi, \ \pm\pi)$ --------------------→ 1M $r = f_{xx} = -sinx - \sin(x+y)$, $s = f_{xy} = -\sin(x+y)$, $t = f_{yy} = -\sin y - \sin(x+y)$ --------------------→ 2 M At $(\pm\pi, \ \pm\pi)$, $rt - s^2 = 0$. It needs further investigation At $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ $rt - s^2 = \frac{9}{4} > 0$ *and* $r < 0$ so f attains its maximum at $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ ----------→ 1M At $\left(-\frac{\pi}{3}, -\frac{\pi}{3}\right)$ $rt - s^2 = \frac{9}{4} > 0$ *and* $r > 0$ so f attains its minimum at $\left(-\frac{\pi}{3}, -\frac{\pi}{3}\right)$ ---- 1 M The maximum and the minimum values are $\frac{3\sqrt{3}}{2}, -\frac{3\sqrt{3}}{2}$
5(a)	**Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dzdxdy$.** $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dzdxdy = \int_0^1 \int_{y^2}^1 x \, (z)_0^{1-x} \, dx \, dy$ ------------→ 1M $= \int_0^1 \int_{y^2}^1 x(1-x) \, dx \, dy$ ------------→ 1M $= \int_0^1 \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_{y^2}^1 dy$ ------------→ 1M $= \int_0^1 \frac{1}{2}(1-y^4) - \frac{1}{3}(1-y^6) dy$ ------------→2M $= \frac{4}{35}$ ------------→ 2M
5(b)	**Change the order of integration and evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} xy \, dydx$** Over the given region x varies from $\frac{y^2}{4}$ to $2\sqrt{y}$ y varies from 0 to 4 --------→ 2M +1 M (Including diagram)  $\int_0^4 \int_{y^2/4}^{2\sqrt{y}} xy \, dy \, dx = \int_0^4 y \left[\frac{x^2}{2}\right]_{\frac{y^2}{4}}^{2\sqrt{y}} dy$ $= \int_0^4 y \left[4y - \frac{y^4}{16}\right] dy$ ----------→ 2 M $= \frac{64}{3}$ ----------→ 2 M
6(a)	**Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$by changing into polar co-ordinates** Put $x = rcos\theta, y = r \, sin\theta$ therefore $x^2 + y^2 = r^2$ and $J\left(\frac{x,y}{r,\theta}\right) = r$ So replace $dxdy$ by $rdrd\theta$ ------------→ 2M Over the given region R : r varies from 0 to ∞ θ varies from 0 to $\frac{\pi}{2}$ ------------→ 2M

	$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dxdy = \int_0^{\frac{\pi}{2}} \int_0^\infty e^{-r^2}\, r\, drd\theta$ ------------→ 1M $= \frac{\pi}{4}$ ------------→ 2M
6(b)	**Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.** Area bounded in the xy- plane is $\iint_R dx\, dy$ ------------→ 1M Over the given region x varies from $\frac{y^2}{4a}$ to $2\sqrt{ay}$ y varies from 0 to $4a$ --------→ 2M (Including diagram) $\int_0^{4a} \int_{y^2/4a}^{2\sqrt{ay}} dy\, dx = \int_0^{4a} \left(2\sqrt{ay} - \frac{y^2}{4a}\right) dy$ ----------→ 2 M $= \frac{16a^2}{3}$ ----------→ 2 M
7(a)	**Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$** Given equation is $(3x^2y^4 + 2xy)\, dx + (2x^3y^3 - x^2)\, dy = 0$ $\frac{\partial M}{\partial y} = 12x^2y^3 + 2x,\ \frac{\partial N}{\partial x} = 6x^2y^3 - 2x$ $\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ so given equation is not exact ----------------→ 1M $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{-2}{y} = g(y)$, so $I.F = e^{\int g(y)dy} = \frac{1}{y^2}$ ----------------→ 2M Multiplying given equation by I.F on both sides weget $\left(3x^2y^2 + \frac{2x}{y}\right) dx + \left(2x^3y - \frac{x^2}{y^2}\right) dy = 0$ ----------------→ 1M G.S is $\int_{y\ const} M_1\, dx + \int (terms\ of\ N_1\ not\ containing\ x)\ dy = c$ ---------------→ 1M $x^3y^2 + \left(\frac{x^2}{y}\right) = c$ ----------------→ 2M
7(b)	**A hot coal (temperature 120^0C) is immersed in ice water (temperature 0^0C). After 20 seconds the temperature of the coal drops to 80^0C. Assume that the ice water is kept at 0^0C. When does the temperature of the coal will reach to 20^0C.** Statement: Newtons Law of cooling -----------------→ 1 M $\theta = \theta_0 + ce^{-kt}$ -----------------→ 1 M $c = 120, k = -\frac{1}{20} ln\left(\frac{2}{3}\right)$ -----------------→ 3 M When $\theta = 20c^0$ $\quad ln\left(\frac{1}{6}\right) = \frac{1}{20} ln\left(\frac{2}{3}\right) t$ -----------------→ 1 M $\therefore t = 88.38\ secs$ -----------------→ 1 M

8 (a)	**Solve the differential equation $(D^3-3D^2+4D-2)y=e^x+cosx$** $y_c=c_1e^x+e^x(c_2\cos x+c_3\ sinx)$ where c_1,c_2,c_3 are arbitrary constants ---------→ 2 M $y_p=\frac{1}{D^3-3D^2+4D-2}e^x+\frac{1}{D^3-3D^2+4D-2}\cos x=I_1+I_2$ $I_1=\frac{1}{D^3-3D^2+4D-2}e^x=\frac{xe^x}{3D^2-6D+4}=\frac{xe^x}{1}$ --------------------→ 2 M $I_2=\frac{1}{D^3-3D^2+4D-2}\cos x=\frac{1}{10}[3\sin x+\cos x]$ ------------------→ 2 M G.S is $y=y_c+y_p=c_1e^x+e^x(c_2\cos x+c_3\ sinx)+xe^x+\frac{1}{10}[3\sin x+\cos x]$ --→ 1 M
8(b)	**Using the method of variation of parameters, solve $(D^2+4)y=sec2x$** $y_c=(c_1\cos 2x+c_2\sin 2x)=c_1u+c_2v$ ------------------→ 2 M Where $u=\cos 2x$; $v=sin\ 2x$ $w=\begin{vmatrix}u & v\\ u' & v'\end{vmatrix}=2$ --------------------→ 1 M $y_p=A\ cos\ 2x+B\ sin\ 2x$ $A=-\int\frac{vR}{w}dx;\quad B=\int\frac{uR}{w}dx$ --------------------→ 1 M A $=\frac{\ln cos2x}{4}$; $B=\frac{x}{2}$ --------------------→ 2 M $\therefore\ y_p=cos2x\frac{\ln cos2x}{4}+\frac{x}{2}\ sin2x$ Hence the GS is $y=y_c+y_p=(c_1\cos 2x+c_2\sin 2x)+cos2x\frac{\ln cos2x}{4}+\frac{x}{2}\ sin2x$ ---→ 1 M
9(a)	**Find the Laplace transform of $\frac{e^{-at}-e^{-bt}}{t}$** Let $f(t)=e^{-a}-e^{-bt}$ $\therefore\ L[f(t)]=L[e^{-at}-e^{-bt}]=\frac{1}{s+a}-\frac{1}{s+b}=F(s)$ ------------------→ 2 M We know that $L\left[\frac{f(t)}{t}\right]=\int_s^\infty F(s)ds=\int_s^\infty\left(\frac{1}{s+a}-\frac{1}{s+b}\right)ds$ ------------------→ 2 M $=ln\left(\frac{s+b}{s+a}\right)$ ------------------→ 3 M
9(b)	**Find inverse Laplace transform of $F(s)=tan^{-1}\left(\frac{s}{2}\right)$** Given $F(s)=tan^{-1}\left(\frac{s}{2}\right)$ $\therefore\ \frac{d[F(s)]}{ds}=\frac{2}{s^2+4}$ ------------------→ 2 M $L^{-1}\left[\frac{d[F(s)]}{ds}\right]=L^{-1}\left[\frac{2}{s^2+4}\right]=sin2t$ ------------------→ 2 M We know that $L^{-1}[F(s)]=\frac{-1}{t}L^{-1}\left[\frac{d[F(s)]}{ds}\right]$ ------------------→ 2 M $\therefore\ L^{-1}\left[tan^{-1}\left(\frac{s}{2}\right)\right]=\frac{-sin2}{t}$ ----------------→1 M

10 (a)	**Using Laplace Transforms, solve the initial value problem** $\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0, given y(0) = 1, y'(0) = y''(0) = 2$ $[s^3Y(s) - s^2y(0) - sy'(0) - y''(0)] + 2[s^2Y(s) - sy(0) - y'(0)] - [sY(s) - y(0)] - 2Y(s) = 0$ ------------------→ 1 M $Y(s) = \frac{s^2+4s+5}{(s^3+2s^2-s-2)}$ ------------------→ 2 M $Y(s) = \frac{5}{3(s-1)} - \frac{1}{(s+1)} + \frac{1}{3(s+2)}$ ------------------→ 2 M Apply ILT on both sides, we get $y(t) = \frac{5}{3}e^{t} - e^{-t} + \frac{1}{3}e^{-2t}$ ------------------→ 2 M
10(b)	**Find the Laplace Transform of** $f(t) = e^{-2t}sin3tcos2t$ WKT $sin3tcos2t = \frac{1}{2}[sin\ 5t + sin\ t]$ $L[sin3tcos2t] = \frac{1}{2}\{L[\sin 5t] + L[\sin t]\} = \frac{1}{2}\left\{\frac{5}{s^2+25} + \frac{1}{s^2+1}\right\} = F(s)$ ------------------→ 2 M First Shifting Property ------------------→ 2 M $L[e^{-2t}sin3tcos2t\ ] = \frac{1}{2}\left\{\frac{5}{(s+2)^2+25} + \frac{1}{(s+2)^2+1}\right\}$ ------------------→ 3 M

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