Subject Code: 24BM11RC02 **R-24** **Reg No:** | | | | | | | | | | | | |

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN
(Autonomous)
(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. - II Semester Regular / Supplementary Examinations, July – 2026

LINEAR ALGEBRA AND VECTOR CALCULUS

[Common to CSE, CSE-AIML, CSE-CS, ECE, EEE and IT]

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

UNIT-I

1. a. Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by reducing to an echelon form **[7M]**

   b. Investigate for what values of λ and μ the system of equations. **[7M]**

   2x + 3y + 5z = 9; 7x + 3y - 2z = 8; 2x + 3y + λz = μ will have

   i) no solution ii) a unique solution and iii) an infinite number of solutions

OR

2. a. Solve the following system of equations by using Gauss elimination method $2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16.$ **[7M]**

   b. Apply Factorization method to solve the equations **[7M]**
   $3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7.$

UNIT-II

3. a. Find the Eigen values and Eigen vectors of a matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **[7M]**

   b. Verify Cayley–Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$ and find inverse of the matrix. **[7M]**

OR

4. a. Find the Eigen values and Eigen vectors of A^{-1} $if\ A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$ **[7M]**

   b. Find the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$ **[7M]**

UNIT-III

5. Reduce the following quadratic form to canonical form by orthogonal transformation. Also, find the rank, index, nature of the quadratic form: **[14M]**
   $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_1x_3.$

OR

6. Reduce the Matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the Diagonal form and hence find A^4. **[14M]**

UNIT-IV

7. a. Show that $\bar{f} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ is irrotational and find the scalar potential φ such that $\bar{f} = \nabla\varphi$.

b. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line $\overline{PQ}$ where $Q(5,0,4)$. **[7M]**

OR

8. a. Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z = 47$ at $(4, -3, 2)$. **[7M]**

b. Show that $\nabla^2 r^n = n(n+1)r^{n-2}$.

UNIT-V

9. a. Find the work done by the force $\vec{F} = (2y+3)\hat{\imath} + zx\hat{\jmath} + (yz - x)\hat{k}$ when it moves a particle from the point $(0,0,0)$ to $(2,1,1)$ along the curve $x = 2t^2, y = t, z = t^3$. **[7M]**

b. Evaluate by Green's theorem $\oint (y - sinx)dx + cosx\, dy$, where C is the rectangle enclosed by the lines $y = 0, x = \frac{\pi}{2}, \pi y = 2x$. **[7M]**

OR

10. a. Use Stoke's theorem evaluate the integral $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = (2y^2)\hat{\imath} + 3x^2\hat{\jmath} - (2x+z)\hat{k}$ and C is the boundary of the triangle whose vertices are (0, 0, 0), (2, 0, 0), (2, 2, 0). **[7M]**

b. Use Divergence theorem to evaluate $\iint_S \vec{F}.dS$ where $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} - z^2\hat{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4, z = 0$ and $z = 3$. **[7M]**