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**GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN**  
(AUTONOMOUS)

(Affiliated to Andhra University, Visakhapatnam)

**B.Tech. - I Semester Regular Examinations, December / January – 2025**

**CALCULUS AND DIFFERENTIAL EQUATIONS**

(Common to <<CSE, CSE-AIML, IT, ECE and EEE>>)

- All questions carry equal marks
- Must answer all parts of the question at one place

**Time: 3Hrs.**

**Max Marks: 70**

**UNIT-I**

- If  $u = (x^2 + y^2 + z^2)^{-1/2}$ , Then prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$  [7]
  - Calculate  $\frac{\partial(u,v,w)}{\partial(x,y,z)}$  if  $u = x^2 - 2y$ ,  $v = x + y + z$ ,  $w = x - 2y + 3z$ . [7]

OR

- If  $u = \sin^{-1}(x - y)$ ,  $x = 3t$  and  $y = 4t^3$ , Show that  $\frac{du}{dt} = 3(1 - t^2)^{-1/2}$ . [7]
  - Determine the Taylor's series expansion of  $f(x, y) = x^2 + 3y^2 - 9x - 9y + 26$  about the point  $(2, 2)$ . [7]

**UNIT-II**

- Discuss the Maxima and Minima of  $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$  [7]
  - Discuss the maxima, minima of the function  $f = x^2 + y^2 + z^2$  where  $x, y, z$  are connected by the relation  $xyz = 216$ . [7]

OR

- Examine the function  $x^3 + y^3 - 3axy$  for the maxima and minima. [7]
  - Find the minimum value of  $x^2 + y^2 + z^2$  having given  $ax + by + cz = p$ . [7]

**UNIT-III**

- Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx dy dz$ . [7]
  - By applying the Change of order of integration evaluate  $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$  [7]

OR

- Evaluate  $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$ . [7]
  - Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and planes  $y + z = 4$  and  $z = 0$ . [7]

**UNIT-IV**

7. a. Solve  $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$  [7]

b. Apply the method of variation of parameters to solve  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$  [7]

OR

8. a. Solve  $(D^2 - 1)y = e^x + x^2e^x$ . [7]

b. A body originally at  $80^\circ\text{C}$  cools down to  $60^\circ\text{C}$  in 20 minutes, the temperature of the air being  $40^\circ\text{C}$ . What will be the temperature of the body after 40 minutes from the original? [7]

### UNIT-V

9. a. Find the Laplace transform of  $\frac{\cos at - \cos bt}{t}$ . [7]

b. Solve  $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}$ ;  $y(0) = 1, y'(0) = 1$ , at  $t = 0$  by using Laplace transform method. [7]

OR

10. a. Evaluate  $\int_0^\infty te^{-2t} \sin 3t dt$  using Laplace transform. [7]

b. Find  $L^{-1} \left[ \frac{1}{(s^2+1)(s^2+9)} \right]$  using convolution theorem. [7]