UNIT-I

ECONOMIC OPERATION OF POWER SYSTEM-1

1.1 HEAT RATE CURVE:

The heat rate characteristics obtained from the plot of the net heat rate in Btu/kWh or kcal/kWh versus power output in kW is shown in fig.1

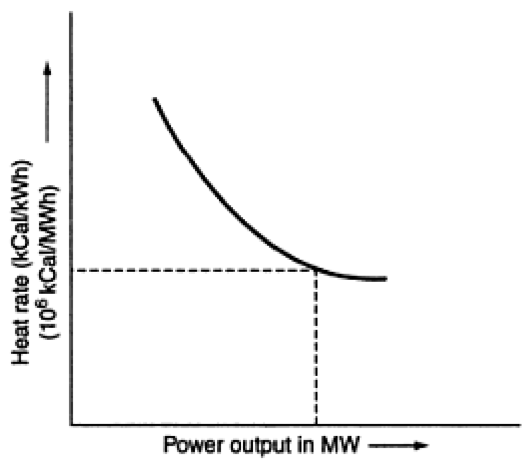


Fig.1. heat rate curve

The thermal unit is most efficient at a minimum heat rate, which corresponds to a particular generation $P_{G.}$ The curve indicates an increase in heat rate at low and high power limits.

Thermal efficiency of the unit is affected by following factors:

Condition of steam

System cycle used

Re-heat stages

Condenser pressure,etc.

1.2 COST CURVES:

To convert the input-output curves into cost curves, the fuel input per hour is multiplied with the cost of the fuel(expressed on Rs./million kCal).

i.e., $(kCal \times 10^6)$/ hr×Rs./million kCal

= million kCal/hr × Rs./million kCal

=Rs./hr

1.3 INCREMENTAL FUEL COST CURVE:

From the input –output curves, the incremental fuel cost (IFC) curve can be obtained. The IFC is defined as the ratio of a small change in the input to the corresponding small change in the output.

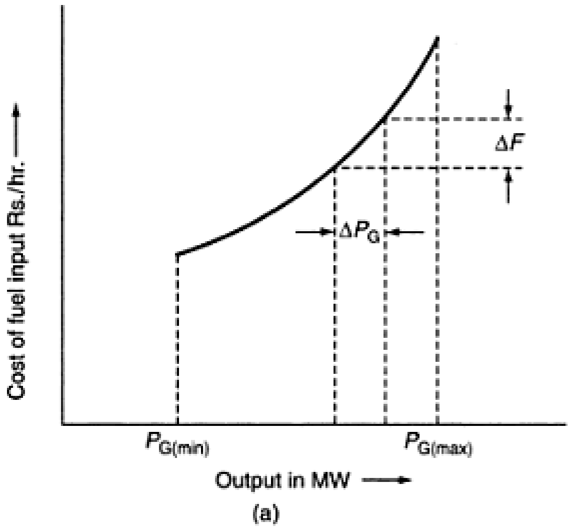
Incremental fuel cost = Δ input / Δ output

` $= \Delta F / \Delta P_G$

Where Δ represents small changes.

As the Δ quantities become progressively smaller, it is seen that the IFC is d(input)/d(output) and is expressed in Rs./MWh. A typical plot of IFC versus output power is shown in fig(a).

The incremental cost curve is obtained by considering the change in the cost of the generation to the change in real-power generation at various points on the input –output curves, i.e., slope of the input-output curve as shown in fig(b).



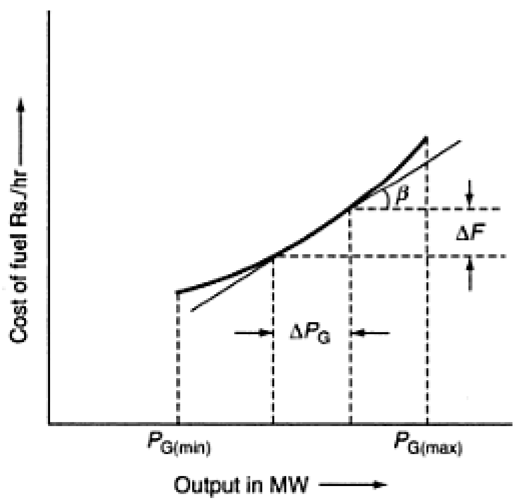


Fig: a) incremental cost curve, (b) incremental fuel cost characteristics in terms of the slope of the input-output curve

The IFC is now obtained as

$(IC)_i$ = slope of the fuel cost curve

i.e., $\tan\beta = \Delta F / \Delta P_G$ in Rs./MWh.

The IFC (IC) of the i^{th} thermal unit is defined, for a given power output, as the limit of the ratio of the increased cost of fuel input (Rs./hr) to the corresponding increase in power output (MW), as the increasing power output approaches zero.

i.e., (IC) = $\lim_{PG_i \to 0} \left(\frac{\partial F_i}{\partial PG_i} \right)$

$$= \left(\frac{dF_i}{dP_{Gi}} \right)$$

$(IC)_i = \left(\frac{dC_i}{dP_{Gi}} \right)$ [$\left(\frac{dF_i}{dP_{Gi}} \right) = \left(\frac{dC_i}{dP_{Gi}} \right) =$ Incremental fuel cost of the i^{th} unit]

Where C_i is the cost of fuel of the i^{th} unit and P_{Gi} is the power generation output of that i^{th} unit.

Mathematically the IFC curve expression can be obtained from the expression of the cost curve.

1.4 INCREMENTAL PRODUCTION COST:

The incremental production cost of a given unit is made up of the IFC plus the incremental cost of items such as labor, supplies, maintenance, and water.

It is necessary for a rigorous analysis to be able to express the costs of these production items as a function of output. However, no methods are presently available for expressing the cost of labor, supplies, or maintenance accurately as a function of output.

Arbitrary methods of determining the incremental costs of labor, supplies, and maintenance are used, the commonest of which is to assume these costs to be a fixed percentage of the IFCs.

In many systems, for purposes of scheduling generation, the incremental production cost is assumed to be equal to the IFC.

1.5 Mathematical determination of optimal allocation of total load among different units:

Consider a power station having 'n' number of units. Let us assume that each unit does not violate the inequality constraints and let the transmission losses be neglected.

The cost of production of electrical energy

$$C = \sum_{i=1}^{n} C i(P_{GI}) \qquad \text{.......(i)}$$

Where Ci is the cost function of the i^{th} unit.

This cost is to be minimized to the equality constraint given by

$$P_D = \sum_{i=1}^{n} P_{GI}$$

$$\text{Or} \quad \sum_{i=1}^{n} P_{GI} - P_D \qquad \text{........(ii)}$$

Where P_{Gi} is the real power generation of the i^{th} unit.

This is a constrained optimization problem.

To get the solution for the optimization problem, we will define an objective function by augmenting equation(i) with an equality constraint equation(ii) through the legrangian multiplier(λ) as

$$\acute{C} = C - \lambda \sum_{i=1}^{n} P_{GI} - P_D]$$

¿

$$\text{Min}[\acute{c}] = \min[C-\lambda[\sum_{i=1}^{n} P_{GI} - P_D]] \qquad \text{........(iii)}$$

The condition for optimality of such an augmented objective function is

$$\frac{\partial \acute{C}}{\partial P_{GI}} = 0$$

From equation (iii)

$$\frac{\partial \acute{C}}{\partial P_{GI}} = \frac{\partial C}{\partial P_{GI}} - \frac{\partial}{\partial P_{GI}} (\lambda[\sum_{i=1}^{n} P_{GI} - P_D]) = 0$$

i.e., $\frac{\partial \acute{C}}{\partial P_{GI}} = \frac{\partial C}{\partial P_{GI}} - (\lambda[1-0])$

since P_D is a constant and is an uncontrolled variable, $\frac{\partial P D}{\partial P_{GI}} = 0$

$$\frac{\partial \acute{C}}{\partial P_{GI}} = \frac{\partial C}{\partial P_{GI}} - \lambda = 0$$

Or $\frac{\partial C}{\partial P_{G1}} - \lambda = 0$

$$\frac{\partial C}{\partial P_{G2}} - \lambda = 0$$

$$\frac{\partial C}{\partial P_{G3}} - \lambda = 0$$

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$$\frac{\partial C}{\partial P_{Gn}} - \lambda = 0$$

There fore $\frac{\partial C}{\partial P_{G1}} = \frac{\partial C}{\partial P_{G2}} = \frac{\partial C}{\partial P_{G3}} = \ldots\ldots = \frac{\partial C}{\partial P_{Gn}} = \lambda$

…………(iv)

Since the expression of C is in a variable separable form, i.e., the overall cost is the summation of cost of each generating unit, which is a function of real-power generation of that unit only:

i.e., $\frac{\partial C}{\partial P_{G1}} = \frac{\partial C1}{\partial P_{G1}}$

$$\frac{\partial C}{\partial P_{G2}} = \frac{\partial C2}{\partial P_{G2}}$$

$$\frac{\partial C}{\partial P_{G3}} = \frac{\partial C3}{\partial P_{G3}}$$

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. $\frac{\partial C}{\partial PGn} = \frac{\partial Cn}{\partial PGn}$

Therefore $\frac{\partial C1}{\partial P_{G1}} = \frac{\partial C2}{\partial P_{G2}} = \frac{\partial C3}{\partial P_{G3}} = \ldots\ldots = \frac{\partial Cn}{\partial P_{Gn}} = \lambda$ ………(v)

In equation (v) each of these derivatives represents the individual incremental cost of every unit. Hence, the condition for the optimal allocation of the total load among the various units, when neglecting the transmission losses, is that the incremental costs of the individual units are equal. It is called a co-ordination equation.

Assume that one unit is operating at a higher incremental cost than the other units. If the output power of that unit is reduced and transferred to units with lower incremental operating costs, then the total operating cost decreases. That is, reducing the output of the unit with the higher incremental cost results in a more decrease in cost than the increase in cost of adding the same output reduction to units with lower incremental costs. Therefore, all units must run with same incremental operating costs.

After getting the optimal solution, in the case that the generation of any one unit is below its minimum capacity or above its maximum capacity, then its generation becomes the corresponding limit. For example, if the generation of any unit violates the minimum limit, then the generation of that unit is set at its minimum specified limit and vice versa. Then, the remaining demand is allocated among the remaining units as for the above criteria.

In the solution of an optimization problem without considering the transmission losses, we make use of equal incremental costs, i.e., the machines are so loaded that the incremental cost of production of each machine is the same.

Optimum Generation Scheduling (when line losses are accounted)

From the unit commitment table of a given plant, the fuel cost curve of the plant can be determined in the form of a polynomial of suitable degree by the method of least squares fit. If the transmission losses are neglected, the total system load can be optimally divided among the various generating plants using the equal incremental cost criterion. It is, however, unrealistic to neglect transmission losses particularly when long distance transmission of power is involved.

A modem electric utility serves over a vast area of relatively low load density. The transmission losses may vary from 5 to 15% of the total load, and therefore, it is essential to account for losses while developing an economic load dispatch policy. It is obvious that when losses are present, we can no longer use the simple 'equal incremental cost' criterion. To illustrate the point, consider a two-bus system with identical generators at each bus (i.e. the same IC curves). Assume that the load is located near plant 1 and plant 2 has to deliver power via a loss line. Equal incremental cost criterion would dictate that each plant should carry half the total load; while it is obvious in this case that the plant 1 should cane a greater share of the load demand thereby reducing transmission losses.

In this section, we shall investigate how the load should be shared among various plants, when line losses are accounted for. The objective is to minimize the overall cost of generation at any time under equality constraint of meeting the load demand with transmission loss, i.e.

$$C=\sum_{i=1}^{k} C_i(P_{Gi}) (2.1)$$

$$\sum_{i=1}^{k} P_{Gi}-P_D-P_L=0 (2.2)$$

where

k = total number of generating plants

P_{Gi} = generation of i_{ih} plant

P_D = sum of load demand in all buses (system load demand)

P_L = total system transmission loss

To solve the problem, we write the Lagrangian as

$$L=\sum_{i=1}^{k} C_i(P_{Gi})-\lambda\left[\sum_{i=1}^{k} P_{Gi}-P_D-P_L\right] (2.3)$$

It will be shown later in this section that, if the power factor of load at each bus is assumed to remain constant, the system loss P_L can be shown to be a function of active power generation at each plant, i.e.

$$P_L=P_L(P_{G1},P_{G2},\ldots\ldots\ldots\ldots P_{Gk}) (2.4)$$

Thus in the optimization problem posed above, P_{Gi} (i=1, 2 ...k) are the only control variables.

For optimum real power dispatch,

$$\frac{\partial L}{\partial P_{Gi}}=\frac{d C_i}{d P_{Gi}}-\lambda+\lambda\frac{\partial P_L}{\partial P_{Gi}}=0\, i=1,2,\ldots..k\,(2.5)$$

Rearranging Eq. (2.5) and recognizing that changing the output of only one plant can affect the cost at only that plant, we have

$$\frac{\frac{d C_i}{d P_{Gi}}}{1-\frac{\partial P_L}{\partial P_{Gi}}}=\lambda(2.6)$$

$$\frac{d C_i}{d P_{Gi}}L_i=\lambda, i=1,2,\ldots.k\,(2.7)$$

where

$$L_i=\frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}(2.8)$$

is called the penalty factor of the ith plant.

The Lagrangian multiplier λ is in rupees per megawatt-hour, when fuel cost is in rupees per hour. Equation (2.6) implies that minimum fuel cost is obtained, when the incremental fuel cost of each plant multiplied by its penalty factor is the same for all the plants.

The (k + 1) variables $(P_{G1}, P_{G2}, \ldots\ldots\ldots\ldots P_{Gk}, \lambda)$ can be obtained from k optimal dispatch Eq. (2.6) together with the power balance Eq. (2.2). The partial derivative $\frac{\partial P_L}{\partial P_{Gi}}$ is termed to as the incremental transmission loss (ITL), associated with the ith generating plant.

Equation (2.6) can also be written in the alternative form

$$(IC)_i=\lambda\left[1-(ITL)_i\right] i=1,2,\ldots\ldots k\,(2.9)$$

This equation is referred to as the exact coordination equation.

Thus it is clear that to solve the optimum load scheduling problem, it is necessary to compute ITL for each plant, and therefore we must determine the functional dependence of transmission loss on real powers of generating plants. There are several methods, approximate and exact, for developing a transmission loss model. One of the most important, simple but approximate, methods of expressing transmission loss as a function of generator powers is through B-coefficients. This method is reasonably adequate for treatment of loss coordination in economic scheduling of load between plants. The general form of the loss formula (derived later in this section) using B-coefficients is

$$P_L = \sum_{m=1}^{k} \sum_{n=1}^{k} P_{Gm} B_{mn} P_{Gn} \quad (2.10)$$

Where

P_{Gm} ,P_{Gn} = real power generation at m, nth plants

B_{mn}= loss coefficients which are constants under certain assumed operating conditions

If P_G's are in megawatts, B_{mn} are in reciprocal of megawatts.

Equation (2.10) for transmission loss may be written in the matrix form as

$$P_L = P_G^T B_{mn} P_G \quad (2.11)$$

Where

$$P_G = \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gk} \end{bmatrix} \wedge B = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1k} \\ B_{21} & B_{22} & \dots & B_{2k} \\ \vdots & \vdots & \dots & \vdots \\ B_{k1} & B_{k2} & \dots & B_{kk} \end{bmatrix}$$

It may be noted that B is a symmetric matrix.

For a three plant system, we can waste the expression for loss as

$$P_L = B_{11} P^2{}_{G1} + B_{22} P^2{}_{G2} + B_{33} P^2{}_{G3} + 2 B_{12} P_{G1} P_{G2} + 2 B_{23} P_{G2} P_{G3} + 2 B_{31} P_{G3} P_{G1} \quad (2.12)$$

With the system power loss model as per Eq. (2.10), we can now write

$$\frac{\partial P_L}{\partial P_{Gi}} = \frac{\partial \left[\sum_{m=1}^{k} \sum_{n=1}^{k} P_{Gm} B_{mn} P_{Gn} \right]}{\partial P_{Gi}}$$

$$\frac{\partial P_L}{\partial P_{Gi}} = \frac{\partial \left[\sum_{\substack{n=1 \\ n \neq i}}^{k} P_{Gi} B_{¿} P_{Gn} + \sum_{\substack{m=1 \\ m \neq i}}^{k} P_{Gm} B_{mi} P_{Gi} + P_{Gi} B_{ii} P_{Gi} \right]}{\partial P_{Gi}} (2.13)$$

It may be noted that in the above expression other terms are independent of P_{Gi}, and are, therefore, left out.

Simplifying Eq. (2.13) and recognizing that $B_{ij} = B_{ji}$, we can write

$$\frac{\partial P_L}{\partial P_{Gi}} = \sum_{j=1}^{k} 2 B_{ij} P_{Gj} (2.14a)$$

Assuming quadratic plant cost curves as

$$C_i(P_{Gi}) = \frac{1}{2} a_i P^2{}_{Gi} + b_i P_{Gi} + d_i$$

We obtain the incremental cost as

$$\frac{dC_i}{dP_{Gi}} = a_i P_{Gi} + b_i (2.14b)$$

Substituting $\frac{\partial P_L}{\partial P_{Gi}}$ and $\frac{dC_i}{dP_{Gi}}$ from above in the coordination Eq. (2.5), we have

$$a_i P_{Gi} + b_i + \lambda \sum_{j=1}^{k} 2 B_{ij} P_{Gj} = \lambda (2.15)$$

Collecting all terms of P_{Gi} and solving for P_{Gi}, we obtain

$$2 B_{ij} P_{Gj} - b_i + ¿ \lambda$$

$$(a_i + 2 \lambda B_{ii}) P_{Gi} = -\lambda \sum_{\substack{j=1 \\ j \neq i}}^{k} ¿$$

$$P_{Gi} = \frac{1 - \frac{b_i}{\lambda} - \sum_{\substack{j=1 \\ j \neq i}}^{k} 2 B_{ij} P_{Gj}}{\frac{a_i}{\lambda} + 2 B_{ii}} ; i = 1, 2, \ldots, k (2.16)$$

For any particular value of λ Eq. (2.16) can be solved iteratively by assuming initial values of P_{Gi}'s (a convenient choice is $P_{Gi} = 0$; i = l, 2... k). Iterations are stopped when P_{Gi}'s converge within specified accuracy.

Derivation of Transmission Loss Formula

The aim of this article is to give a simpler derivation by making certain assumptions. Figure 2.1 (c) depicts the case of two generating plants connected to an arbitrary number of loads through a transmission network. One line within the network is designated as branch p.

Imagine that the total load current I_D is supplied by plant 1 only, as in Fig. 2.1(a). Let the current in line p be I_{P1}. Define

$$M_{P1}=I_{p1}/I_D \qquad (2.17)$$

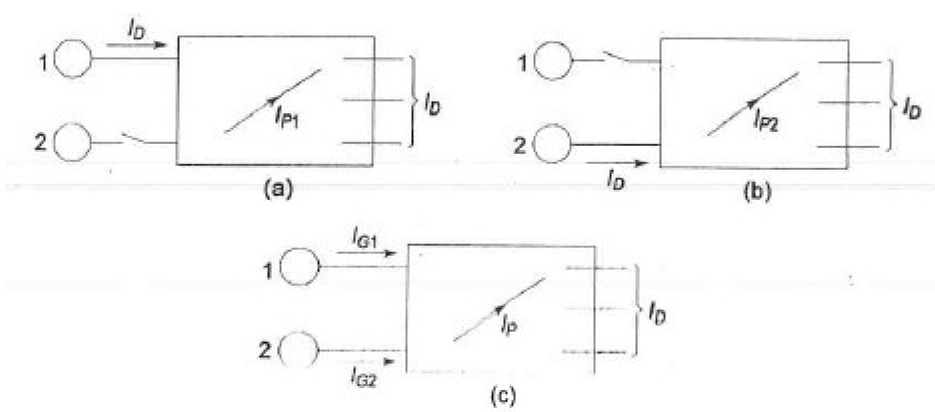

Fig. 2.1 Schematic diagram showing two plants connected through a power network to a number of loads

Fig. 2.1 Schematic diagram showing two plants connected through a power network to a number of loads

Similarly, with plant 2 alone supplying the total load current (Fig. 2.1b), we can define

$M_{P2}=I_{p2}/I_D$ (2.18)

M_{p1} and M_{p2} are called current distribution factors. The values of current distribution factors depend upon the impedances of the lines and their interconnection and are independent of the current I_D.

When both generators 1 and 2 are supplying current into the network as in Fig. 2.1(c), applying the principle of superposition the current in the line p can be expressed as

$$I_P = M_{P1}\, I_{G1} + M_{P2}\, I_{G2} \qquad (2.19)$$

Where I_{G1} and I_{G2} are the currents supplied by plants 1 and 2, respectively.

At this stage let us make certain simplifying assumptions outlined below:

(1) All load currents have the same phase angle with respect to a common reference. To understand the implications of this assumption consider the load current at the i_{th} bus. It can be written as

$$\delta$$

$$(¿¿i-\varnothing_i)=|I_{Di}|\angle\theta_i$$

$$|I_{Di}|\angle ¿$$

Where δ_i the phase is angle of the bus voltage and $\varnothing_i$ is the lagging phase angle of the load. Since δ_i and $\varnothing_i$ vary only through a narrow range at various buses, it is reasonable to assume that θ_i is the same for all load currents at all times.

(2) Ratio X/R is the same for all network branches.

These two assumptions lead us to the conclusion that I_{P1} and I_D (Fig. 2.1(a)) have the same phase angle and so have I_{P2} and I_D [Fig. 2.1(b)], such that the current distribution factors M_{P1} and M_{P2} are real rather than complex.

Let, $I_{G1}=|I_{G1}|\ \angle\sigma_1$ and $I_{G2}=|I_{G2}|\ \angle\sigma_2$

Where $\angle \sigma_1$ and $\angle \sigma_2$ are phase angles of I_{G1} and I_{G2}, respectively with respect to the common reference.

From Eq. (2.19), we can write

$$\text{¿} I_P \vee \text{¿}^2 = \left(M_{P1}|I_{G1}|\cos\sigma_1 + M_{P2}|I_{G2}|\cos\sigma_2\right)^2 + \left(M_{P1}|I_{G1}|\sin\sigma_1 + M_{P2}|I_{G2}|\sin\sigma_2\right)^2 (2.20)$$

$$\text{¿}$$

Expanding the simplifying the above equation, we get

$$\text{¿} I_P \vee \text{¿}^2 = M^2{}_{P1}|I_{G1}|^2 + M^2{}_{P2}|I_{G2}|^2 + 2M_{P1}M_{P2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)(2.21)$$

$$\text{¿}$$

Now $|I_{G1}| = \frac{P_{G1}}{\sqrt{3}|V_1|\cos\varnothing_1}$; $|I_{G2}| = \frac{P_{G2}}{\sqrt{3}|V_2|\cos\varnothing_2}$

(2.22)

Where P_{G1} and P_{G2} are the three-phase real power outputs of plants 1 and 2 at power factors of cos $\varnothing_1$ and cos $\varnothing_2$, and V_1 and V_2 are the bus voltages at the plants.

If R_p is the resistance of branch p, the total transmission loss is given by

$$P_L = \sum_p 3|I_p|^2 R_p$$

Substituting for $|I_p|^2$ from Eq. (2.21), and $|I_{G1}|$ and $|I_{G2}|$ from Eq. (2.22), we obtain

$$M^2{}_{P1}R_p + \text{¿}\frac{2P_{G1}P_{G2}\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\varnothing_1\cos\varnothing_2}\sum_p M_{P1}M_{P2}R_p + \frac{P_{G2}{}^2}{|V_2|^2(\cos\varnothing_2)^2}\sum_p M^2{}_{P2}R_p (2.23)$$

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2(\cos\varnothing_1)^2}\sum_p \text{¿}$$

Equation (2.23) can be recognized as

$$P_L = P_{G1}{}^2 B_{11} + 2P_{G1}P_{G2}B_{12}B_{12} + P_{G1}{}^2 B_{11}(2.24)$$

$$B_{11}=\frac{1}{|V_1|^2(\cos\varnothing_1)^2}\sum_p M^2{}_{P1}R_p$$

$$B_{12}=\frac{\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\varnothing_1\cos\varnothing_2}\sum_p M_{P1}M_{P2}R_p(2.25)$$

$$B_{22}=\frac{1}{|V_2|^2(\cos\varnothing_2)^2}\sum_p M^2{}_{P2}R_p$$

The terms B_{11}, B_{12} and B_{22} are called loss coefficients or B-coefficients. If voltages are line to line kV with resistances in ohms, the units of B-coefficients are in MW^{-1}. Further, with P_{G1} and P_{G2} expressed in MW, P_L will also be in MW.

The above results can be extended to the general case of k plants with transmission loss expressed as

$$P_L=\sum_{m=1}^{k}\sum_{n=1}^{k}P_{Gm}B_{mn}P_{Gn}(2.26)$$

Where

$$B_{mn}=\frac{\cos(\sigma_m-\sigma_n)}{|V_m||V_n|\cos\varnothing_m\cos\varnothing_n}\sum_p M_{Pm}M_{Pn}R_p(2.27)$$

The following assumptions including those mentioned already are necessary, if B-coefficients are to be treated as constants as total load and load sharing between plants vary. These assumptions are:

1, All load currents maintain a constant ratio to the total current.

2. Voltage magnitudes at all plants remain constant.

3. Ratio of reactive to real power, i.e. power factor at each plant remains constant.

4. Voltage phase angles at plant buses remain fixed. This is equivalent to assuming that the plant currents maintain constant phase angle with respect to the common reference, since source power factors are assumed constant as per assumption 3 above.