LECTURE NOTES

ON

DIGITAL CONTROL SYSTEMS

IV B.TECH(EEE) II SEMESTER (JNTUK R13)

Unit I

INTRODUCTION AND SIGNAL PROCESSING

INTRODUCTION TO DIGITAL CONTROL

A digital control system model can be viewed from different perspectives including control algorithm, computer program, conversion between analog and digital domains, system performance etc. One of the most important aspects is the sampling process level.

In continuous time control systems, all the system variables are continuous signals. Whether the system is linear or nonlinear, all variables are continuously present and therefore known (available) at all times. A typical continuous time control system is shown in Figure 1.

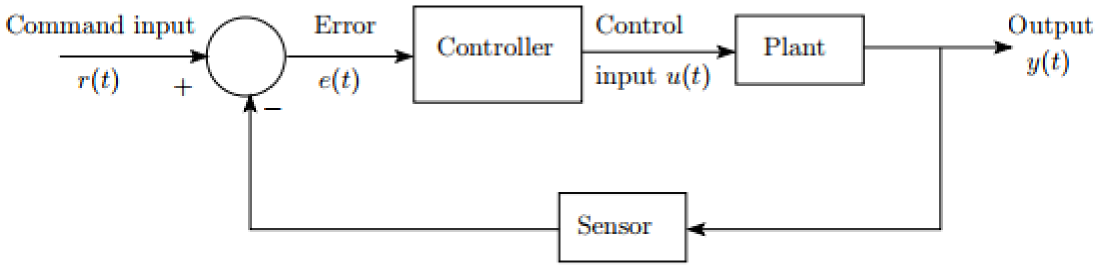


Figure 1: A typical closed loop continuous time control system

In a digital control system, the control algorithm is implemented in a digital computer. The error signal is discretized and fed to the computer by using an A/D (analog to digital) converter. The controller output is again a discrete signal which is applied to the plant after using a D/A (digital to analog) converter. General block diagram of a digital control system is shown in Figure 2.

e(t) is sampled at intervals of T. In the context of control and communication, sampling is a process by which a continuous time signal is converted into a sequence of numbers at discrete time intervals. It is a fundamental property of digital control systems because of the discrete nature of operation of digital computer.

Figure 3 shows the structure and operation of a finite pulse width sampler, where (a) represents the basic block diagram and (b) illustrates the function of the same. T is the sampling period and p is the sample duration.

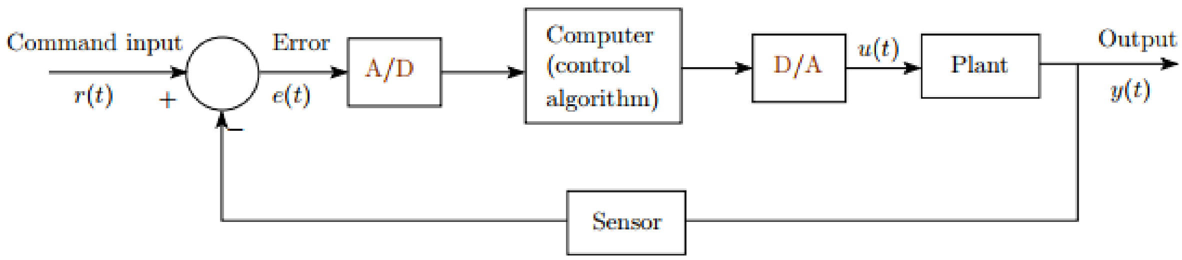


Figure 2: General block diagram of a digital control system

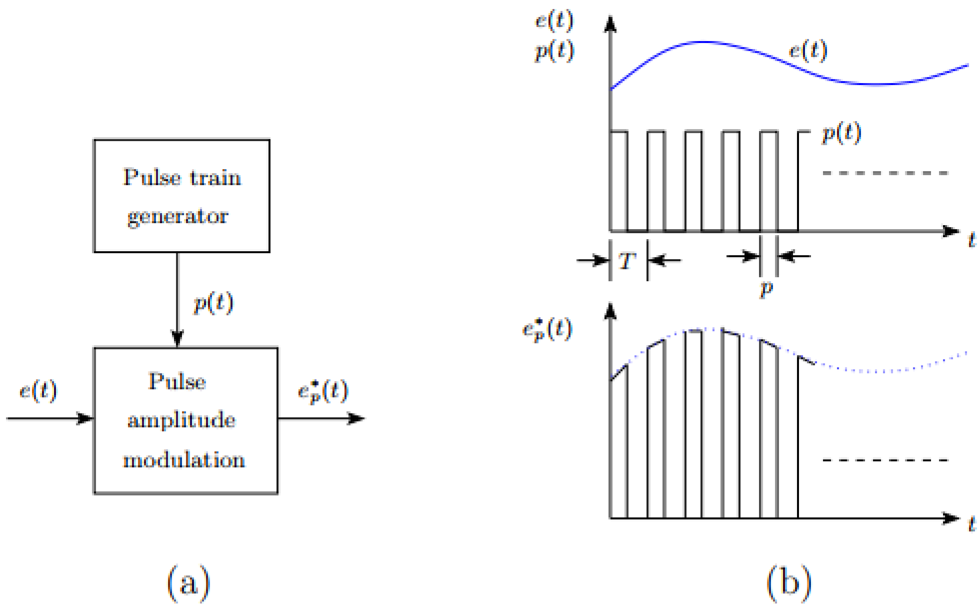


Figure 3: Basic structure and operation of a finite pulse width sampler

In the early development, an analog system, not containing a digital device like computer,in which some of the signals were sampled was referred to as a sampled data system. With the advent of digital computer, the term discrete-time system denoted a system in which all its signals are in a digital coded form. Most practical systems today are of hybrid nature, i.e., contains both analog and digital components.

Before proceeding to any depth of the subject we should first understand the reason behind going for a digital control system. Using computers to implement controllers has a number of advantages. Many of the difficulties involved in analog implementation can be avoided. Few of them are enumerated below.

1. Probability of accuracy or drift can be removed.

2. Easy to implement sophisticated algorithms.

3. Easy to include logic and nonlinear functions.

4. Reconfigurability of the controllers.

HOW WAS THEORY DEVELOPED ?

1. Sampling Theorem: Since all computer controlled systems operate at discrete times only, it is important to know the condition under which a signal can be retrieved from its values at discrete points. Nyquist explored the key issue and Shannon gave the complete solution which is known as Shannon's sampling theorem. We will discuss Shannon's sampling theorem in proceeding lectures.

2. Difference Equations and Numerical Analysis: The theory of sampled-data system is closely related to numerical analysis. Difference equations replaced the differential equations in continuous time theory. Derivatives and integrals are evaluated numerically by approximating them with differences and sums.

3. Transform Methods: Z-transform replaced the role of Laplace transform in continuous domain.

4. State Space Theory: In late 1950's, a very important theory in control system was developed which is known as state space theory. The discrete time representation of state models are obtained by considering the systems only at sampling points.

ADVANTAGES OF DIGITAL SYSTEMS

1. Ease of programmability
The digital systems can be used for different applications by simply changing the program without additional changes in hardware.

2. Reduction in cost of hardware
The cost of hardware gets reduced by use of digital components and this has been possible due to advances in IC technology. With ICs the number of components that can be placed in a given area of Silicon are increased which helps in cost reduction.

3. High speed
Digital processing of data ensures high speed of operation which is possible due to advances in Digital Signal Processing.

4. High Reliability
Digital systems are highly reliable one of the reasons for that is use of error correction codes.

5. Design is easy
The design of digital systems which require use of Boolean algebra and other digital techniques is easier compared to analog designing.

6. Result can be reproduced easily
Since the output of digital systems unlike analog systems is independent of temperature, noise, humidity and other characteristics of components the reproducibility of results is higher in digital systems than in analog systems.

MATHEMATICAL MODELING OF SAMPLING PROCESS

Sampling operation in sampled data and digital control system is used to model either the sample and hold operation or the fact that the signal is digitally coded. If the sampler is used to represent S/H (Sample and Hold) and A/D (Analog to Digital) operations, it may involve delays, finite sampling duration and quantization errors. On the other hand if the sampler is used to represent digitally coded data the model will be much simpler. Following are two popular sampling operations:

1. Single rate or periodic sampling
2. Multi-rate sampling

We would limit our discussions to periodic sampling only.

Finite pluse width sampler

In general, a sampler is the one which converts a continuous time signal into a pulse modulated or discrete signal. The most common type of modulation in the sampling and hold operation is the pulse amplitude modulation.

The symbolic representation, block digram and operation of a sampler are shown in Figure 1. The pulse duration is *p* second and sampling period is *T* second. Uniform rate sampler is a linear device which satisfies the principle of superposition. As in Figure 1, *p*(t) is a unit pulse train with period *T*.

$$p(t) = \sum_{k=-\infty}^{\infty} [u_s(t-kT) - u_s(t-kT-p)]$$

where $U_s(t)$ represents unit step function. Assume that leading edge of the pulse at *t* = 0coincides with *t* = 0. Thus $f_p^*(t)$ can be written as

$$f_p^*(t) = f(t) \sum_{k=-\infty}^{\infty} [u_s(t-kT) - u_s(t-kT-p)]$$

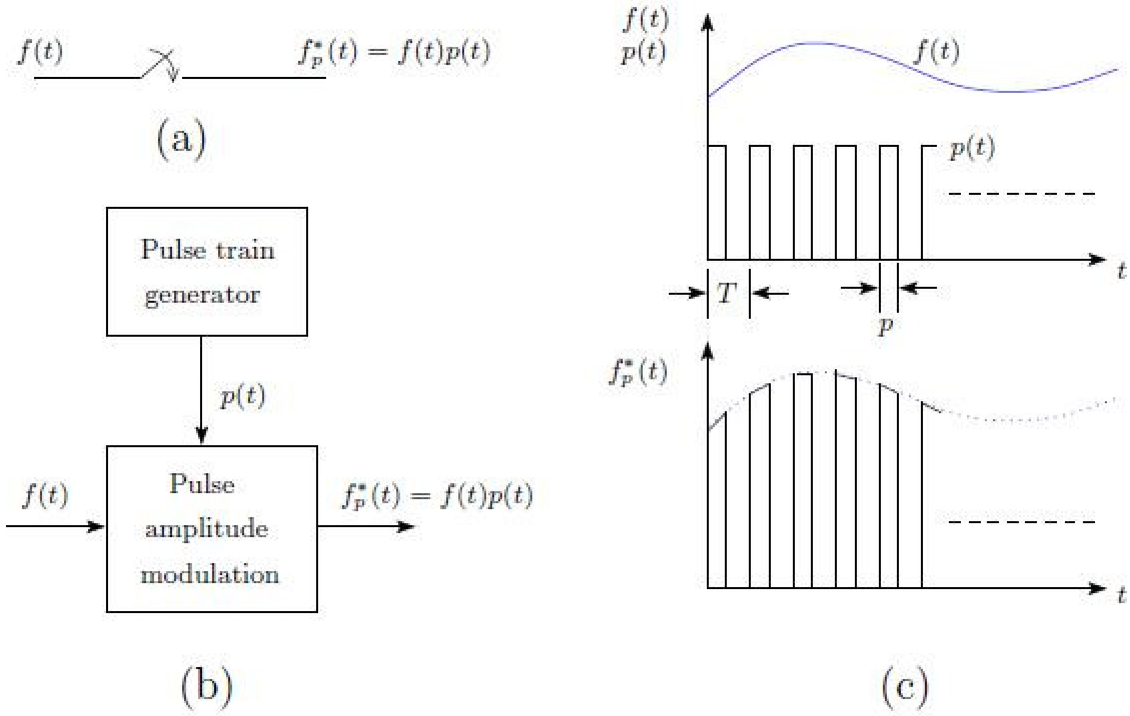


Figure 1: Finite pulse with sampler : (a) Symbolic representation (b) Block diagram (c) Operation

Frequency domain characteristics:

Since $p(t)$ is a periodic function, it can be represented by a Fourier series, as

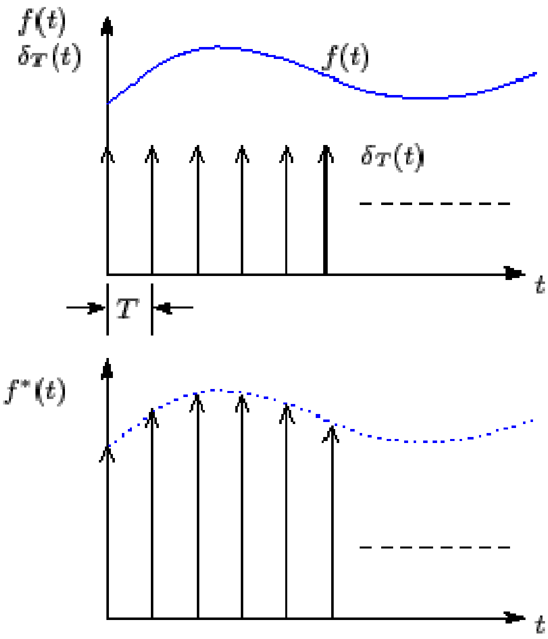
$$p(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnw_s t}$$

where $\omega_s = \frac{2\pi}{T}$ is the sampling frequency and C_n's are the complex Fourier series coefficients.

$$C_n = \frac{1}{T}\int_0^T p(t)e^{-jnw_s t}dt$$

since $p(t) = 1$ for $0 \leq t \leq p$ and 0 for rest of the period.

Ideal Sampler: In case of an ideal sampler, the carrier signal is replaced by a train of unit impulses as shown in Figure 2. The sampling duration *p* approaches 0, i.e., its operation are instantaneous.



The output of an ideal sampler can be expressed as

$$
\begin{aligned}
f^*(t) &= \sum_{k=0}^{\infty} f(kT)\delta(t-kT) \\
\Rightarrow F^*(s) &= \sum_{k=0}^{\infty} f(kT)e^{-kTs}
\end{aligned}
$$

One should remember that practically the output of a sampler is always followed by a hold device which is the reason behind the name sample and hold device. Now, the output of a hold device will be the same regardless the nature of the sampler and the attenuation factor p can be dropped in that case. Thus the sampling process can be be always approximated by an ideal sampler or impulse modulator.

$$
\begin{aligned}
C_n &= \frac{1}{T}\int_0^p e^{-jnw_s t}dt \\
&= \left[\frac{1}{-jnw_s T}.e^{-jnw_s t}\right]_0^p \\
&= \frac{1-e^{-jnw_s p}}{jnw_s T}
\end{aligned}
$$

DATA RECONSTRUCTION

Most of the control systems have analog controlled processes which are inherently driven by analog inputs. Thus the outputs of a digital controller should first be converted into analog signals before being applied to the systems. Another way to look at the problem is that the high frequency components *f*(t) should be removed before applying to analog devices. A low pass filter or a data reconstruction device is necessary to perform this operation.

In control system, hold operation becames the most popular way of reconstruction due to its simplicity and low cost. Problem of data reconstruction can be formulated as: `` Given a sequence of numbers, $f(0), f(T), f(2T), \cdots, f(kt), \cdots$, a continuous time signal $f(t),\ t \geq 0$, is to be reconstructed from the information contained in the sequence."

Data reconstruction process may be regarded as an extrapolation process since the continuous data signal has to be formed based on the information available at past sampling instants. Suppose the original signal *f*(t) between two consecutive sampling instants *kT* and *(k + 1)T* is to be estimated based on the values of *f*(t) at previous instants of *kT*, i.e., *(k - 1)T*, *(k - 2)T*.....0.

Power series expansion is a well known method of generating the desired approximation which yields

$$
\begin{aligned}
f_k(t) &= f(kT) + f^{(1)}(kT)(t-kT) + \frac{f^{(2)}(kT)}{2!}(t-kT)^2 + \ldots.. \\
\text{where, } f_k(t) &= f(t) \text{ for } kT \leq t \leq (k+1)T \text{ and} \\
f^{(n)}(kT) &= \frac{d^n f(t)}{dt^n}\Big|_{t=kT} \quad \text{for } n = 1, 2, \ldots
\end{aligned}
$$

Since the only available information about *f*(t) is its magnitude at the sampling instants, the derivatives of *f*(t) must be estimated from the values of *f*(*kT*), as

$$
\begin{aligned}
f^{(1)}(kT) &\cong \frac{1}{T}[f(kT) - f((k-1)T)] \\
\text{Similarly,} \quad f^{(2)}(kT) &\cong \frac{1}{T}[f^{(1)}(kT) - f^{(1)}((k-1)T)] \\
\text{where,} \quad f^{(1)}((k-1)T) &\cong \frac{1}{T}[f((k-1)T) - f((k-2)T)]
\end{aligned}
$$

Zero Order Hold

Higher the order of the derivatives to be estimated is, larger will be the number of delayed pulses required. Since time delay degrades the stability of a closed loop control system, using higher order derivatives of *f*(t) for more accurate reconstruction often causes serious stability problem. Moreover a high order extrapolation requires complex circuitry and results in high cost.

For the above reasons, use of only the first term in the power series to approximate f(t) during the time interval $kT \leq t < (k+1)T$ is very popular and the device for this type of extrapolation is known as zero-order extrapolator or zero order hold. It holds the value of $f(kT)$ for $kT \leq t < (k+1)T$ until the next sample $f((k+1)T)$ arrives. Figure 1 illustrates the operation of a ZOH where the green line represents the original continuous signal and brown line represents the reconstructed signal from ZOH.

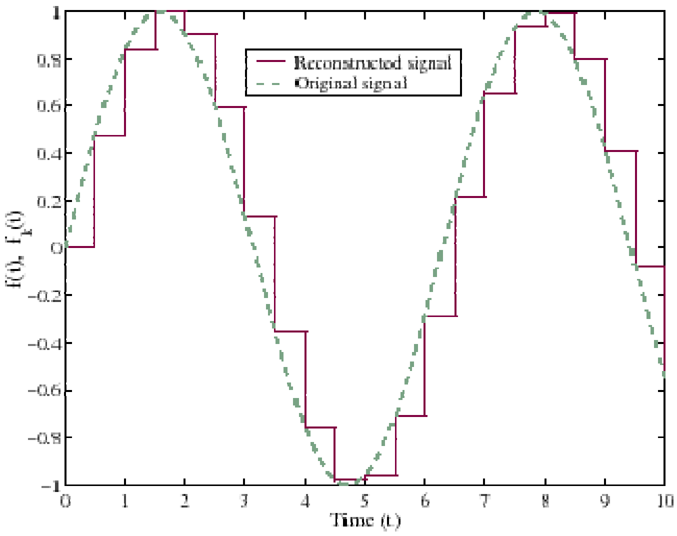


Figure 1: Zero order hold operation

The accuracy of zero order hold (ZOH) depends on the sampling frequency. When $T \to 0$, the output of ZOH approaches the continuous time signal. Zero order hold is again a linear device which satisfies the principle of superposition.

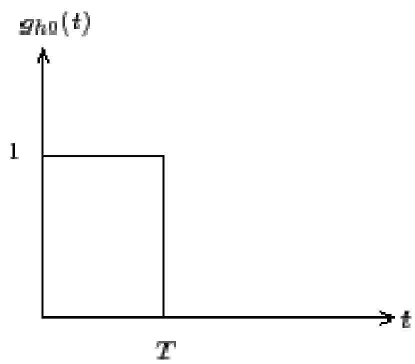


Figure 2: Impulse response of ZOH

The impulse response of a ZOH, as shown in Figure 2, can be written as

$$
\begin{aligned}
g_{ho}(t) &= u_s(t) - u_s(t-T) \\
\Rightarrow G_{ho}(s) &= \frac{1-e^{-TS}}{s} \\
G_{ho}(jw) &= \frac{1-e^{-jwT}}{jw} = T\,\frac{\sin(wT/2)}{wT/2}e^{-jwT/2}
\end{aligned}
$$

Since $T = \frac{2\pi}{w_s}$, we can write

$$G_{ho}(jw) = \frac{2\pi}{w_s}\frac{\sin(\pi w/w_s)}{\pi w/w_s}\,e^{-j\pi w/w_s}$$

Magnitude of $G_{ho}(jw)$:

$$|G_{ho}(jw)| = \frac{2\pi}{w_s}\left|\frac{\sin(\pi w/w_s)}{\pi w/w_s}\right|$$

Phase of $G_{ho}(jw)$:

$$\angle G_{ho}(jw) = \angle \sin(\pi w/w_s) - \frac{\pi w}{w_s} \text{ rad}$$

The sign of $\angle \sin(\pi w/w_s)$ changes at every integral value of $\frac{\pi w}{w_s}$. The change of sign from + to - can be regarded as a phase change of -180° . Thus the phase characteristics of ZOH is linear with jump discontinuities of -180° at integral multiple of ω_s.

The magnitude and phase characteristics of ZOH are shown in Figure 3.

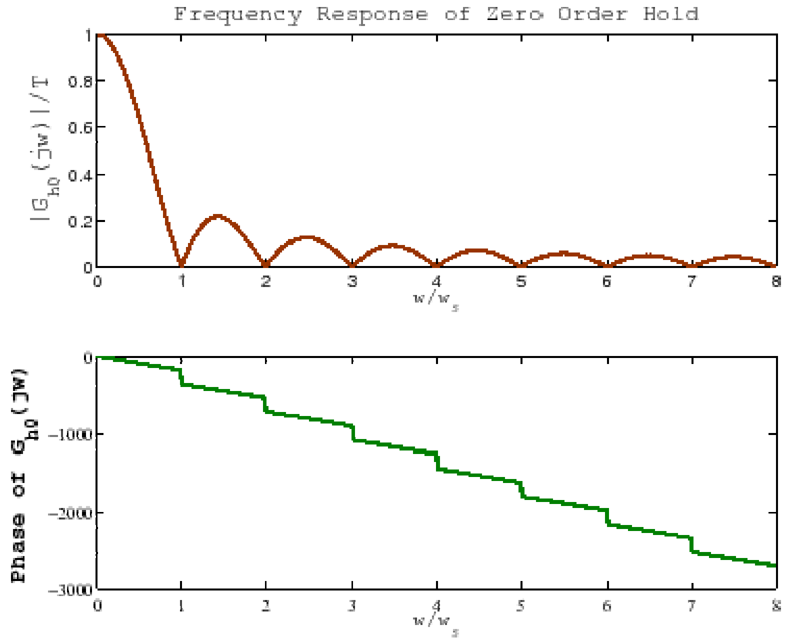


Figure 3: Frequency response of ZOH

At the cut off frequency $w_c = \dfrac{w_s}{2}$, magnitude is 0.636. When compared with an ideal low pass filter, we see that instead of cutting of sharply at $w = \frac{w_s}{2}$, the amplitude characteristics of $G_{ho}(jw)$ is zero at $\omega_s/2$ and integral multiples of ω_s.