LECTURE NOTES

ON

POWER SYSTEM ANALYSIS

III B.TECH(EEE) II SEMESTER (JNTUK R13)

Unit I

PER UNIT REPRESENTATION & TOPOLOGY

Per Unit (pu) System

In power system analysis, it is common practice to use per-unit quantities for analyzing and communicating voltage, current, power, and impedance values. These per-unit quantities are normalized or scaled on a selected base, as shown in the equation below, allowing engineers to simplify power system calculations with multiple voltage transformations

$$per\ unit\ quantity\ = \frac{actual\ quantity}{base\ quantity}$$

Historically, per-unit values have made power calculations performed by hand much simpler. With many calculations now being done using computer software, this is no longer the primary advantage; however, some advantages still exist. For example, when analyzing voltage on a larger system scale with many different nominal voltages via step-up and step-down transformers, per-unit quantities provide an easy way to assess the condition of the entire system without verifying the specific nominal voltage of each subsystem. Another advantage is the fact that per-unit quantities tend to fall in a relatively narrow range, making it easy to identify incorrect data. In addition to these advantages, most power flow analysis software requires input and reports results per unit. For these reasons, it is important for engineers and technicians to understand the per-unit concept.

Understanding Per-Unit Quantities

In three-phase power systems, voltage and apparent power (VA) are typically chosen as bases; from these, current, impedance, and admittance bases can be determined using the following equations.

$$I_{base} = \frac{\text{Apparent Power Base, } VA_{3\phi}}{\sqrt{3} \text{ X Voltage Base, } V_{LL}}$$

$$Z_{base} = \frac{\text{Voltage Base, } V_{LL}}{\sqrt{3} \text{ X Base current (A)}} = \frac{(\text{Voltage Base, } V_{LL})^2}{\text{Apparent Power Base, } VA_{3\phi}}$$

$$Y_{base} = \frac{1}{Z_{base}}$$

For equipment such as motors, generators, and transformers, the base power rating and voltage are typically used to calculate a per-unit impedance. In some instances it is necessary to convert these per-unit values with different power and voltage bases to one common base. The power base will remain constant throughout the system, and the voltage base is typically the nominal voltage for each part of the system. The equation for converting to a new impedance base is as follows:

$$Z_{PU-NEW} = Z_{PU-OLD}\left(\frac{baseV_{OLD}}{baseV_{NEW}}\right)^2\left(\frac{baseVA_{NEW}}{baseVA_{OLD}}\right)$$

Single line diagram or One-line diagram

Electric power systems are supplied by three-phase generators. Ideally, the generators are supplying balanced three phase loads. Fig.1.1 shows a star connected generator supplying star connected balanced load.

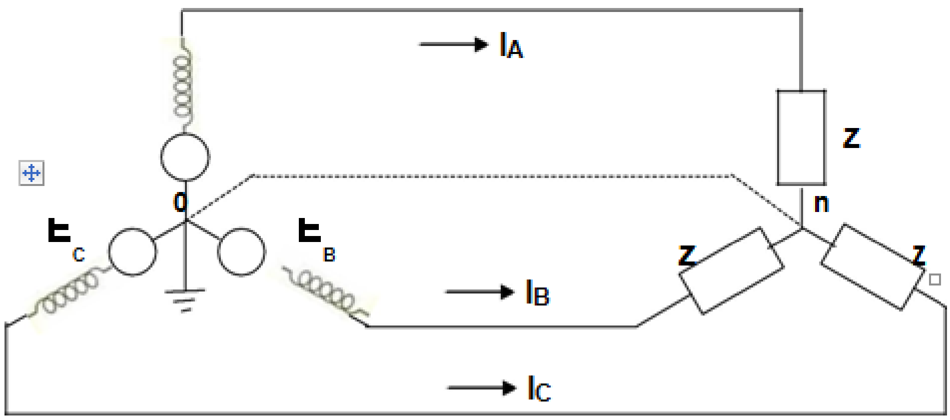


Fig. 1.1 Y- connected generator supplying balanced Y- connected load

A balanced three-phase system is always solved as a single-phase circuit composed of one of the three lines and the neutral return. Single-phase circuit of three-phase system considered above is shown in Fig. 1.2.

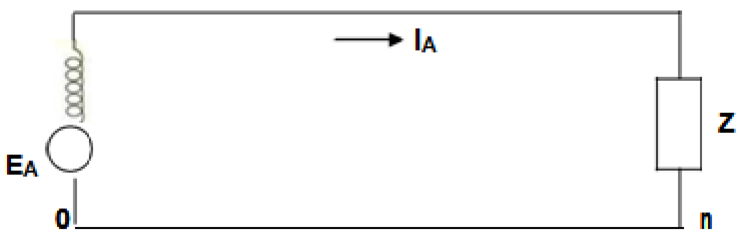


Fig. 1.2 Single-phase circuit

Often the diagram is simplified further by omitting the neutral and by indicating the component parts by standard symbols rather than by their equivalent circuits. Such a simplified diagram of electric system is called a one-line diagram. It is also called as single line diagram. The one-line diagram of the simple three-phase system considered above is shown in Fig. 1.3.

Fig. 1.3 One-line diagram

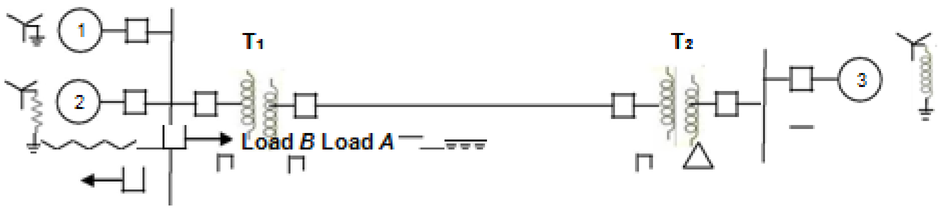


Fig. 1.4 One-line diagram of a sample power system

This system has two generators, one solidly grounded and the other grounded through a resistor, that are connected to a bus and through a step-up transformer to a transmission line. Another generator, grounded through a reactor, is connected to a bus and through a transformer to the other end of the transmission line. A load is connected to each bus.

On the one-line diagram information about the loads, the ratings of the generators and transformers, and reactances of different components of the circuit is often given.

Impedance and reactance diagram

In order to calculate the performance of a power system under load condition or upon the occurrence of a fault, the one line diagram is used to draw the single-phase or per phase equivalent circuit of the system.

Refer the one-line diagram of a sample power system shown in Fig. 1.4.

The impedance diagram does not include the current limiting impedances shown in the one-line diagram because no current flows in the ground under balanced condition.

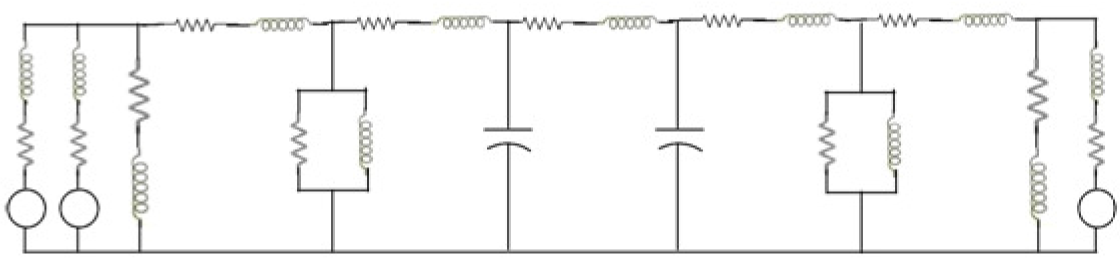


Fig. 1.5 Per-phase impedance diagram

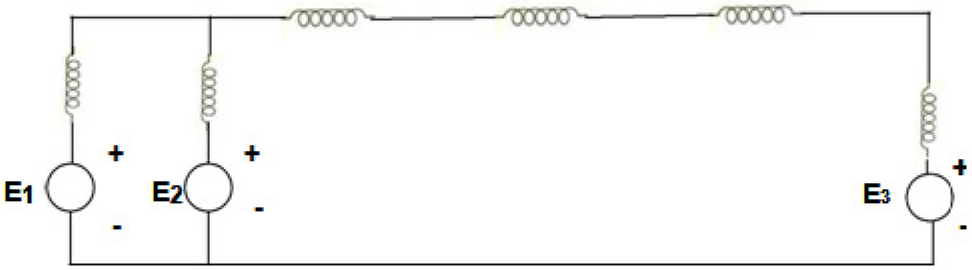


Fig. 1.6 Per-phase reactance diagram

Graph Theory

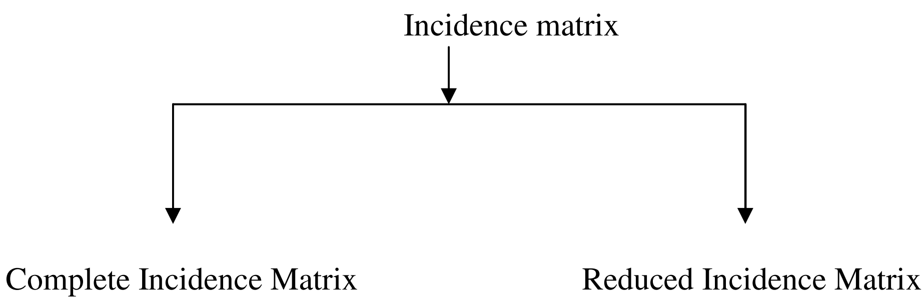
Graph theory is the branch of mathematics dealing with graphs. In network analysis, graphs are used extensively to represent a network being analyzed. The graph of a network captures only certain aspects of a network; those aspects related to its connectivity, or, in other words, its topology.

Incidence matrix:

Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in the graph and the nodes at which this branch is incident. This branch is called incident matrix. When one row is completely deleted from the matrix the remaining matrix is called a reduced incidence matrix.

The rows of the matrix represent the nodes and the columns represent the branches of the graph.

i. The elements of the incidence matrix will be +1, -1 or zero.
ii. If a branch is connected to a node and its orientation is away from the node the corresponding element is marked +1.
iii. If a branch is connected to a node and its orientation is towards the node then the corresponding element is marked – 1.
iv. If a branch is not connected to a given node then the corresponding element is marked zero.



Complete incidence matrix:

An incidence matrix in which the summation of elements in any column is zero is called a complete incidence matrix.

It is an N × B matrix with elements of

$$A_n = [a_{kj}]$$

a_{kj} = 1, when the branch b_j is incident to and oriented away from the k^{th} node.

= −1, when the branch b_j is incident to and oriented towards the k^{th} node.

= 0, when the branch b_j is not incident to the k^{th} node.

As each branch of the graph is incident to exactly two nodes,

$$\sum_{k=0}^{n} a_{kj} = 0$$

That is, each column of A_n has exactly two non zero elements, one being +1 and the other −1. Sum of elements of any column is zero. The columns of A_n are linearly dependent. The rank of the matrix is less than N.

Significance of the incidence matrix lies in the fact that it translates all the geometrical features in the graph into an algebraic expression.

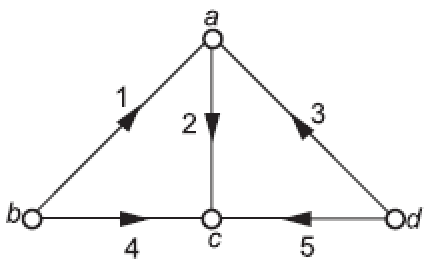
Using the incidence matrix, we can write KCL as

$$\mathbf{A_n} * \mathbf{i_b} = 0,$$

Where $\mathbf{i_b}$ =branch current vector.

But these equations are not linearly independent. The rank of the matrix $\mathbf{A_n}$ is N − 1. This property of $\mathbf{A_n}$ is used to define another matrix called reduced incidence matrix or bus incidence matrix.

For the oriented graph shown in Fig. 2.3(a), the incidence matrix is as follows:



$$A_n = \begin{array}{c} \text{Nodes} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \text{Branches} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Reduced incidence matrix:

The reduced incidence matrix is obtained from a complete incidence matrix by eliminating a row. Hence the summation of elements in any column is not zero.

Any node of a connected graph can be selected as a reference node. Then the voltages of the other nodes (referred to as buses) can be measured with respect to the assigned reference. The matrix obtained from $\mathbf{A_n}$ by deleting the row corresponding to the reference node is the element bus incident matrix **A** and is called bus incidence matrix with dimension $(N-1) \times B$. **A** is rectangular and therefore singular.

From **A**, we have $\mathbf{A} * \mathbf{i_b} = 0$, which represents a set of linearly independent equations and there are $N - 1$ independent node equations.

For the graph shown in Fig 2.3(a), with d selected as the reference node, the reduced incidence matrix is

$$A = \begin{array}{c c} & \text{Branches} \\ \text{Nodes} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

Det AA^T gives the number of all possible trees.

PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cut sets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

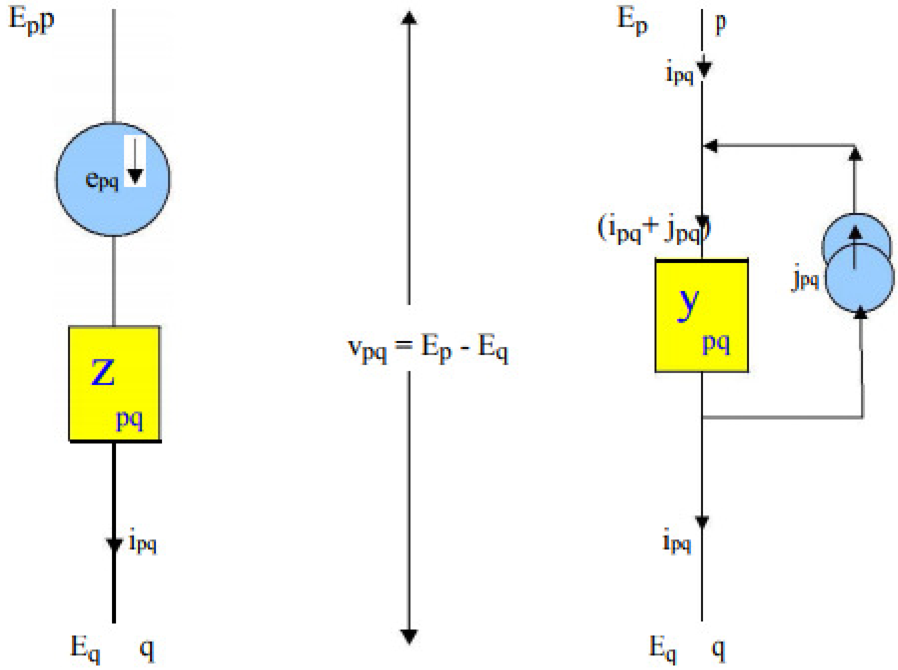


Fig.2 Representation of a primitive network element
(a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

v_{pq} = voltage across the element p-q,
e_{pq} = source voltage in series with the element pq,
i_{pq}= current through the element p-q,
j_{pq}= source current in shunt with the element pq,
z_{pq}= self impedance of the element p-q and
y_{pq}= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

$v_{pq} + e_{pq} = z_{pq}i_{pq}$ (in its impedance form)

$i_{pq} + j_{pq} = y_{pq}v_{pq}$ (in its admittance form)

Thus the parallel source current j_{pq} in admittance form can be related to the series source voltage, e_{pq} in impedance form as per the identity:

$j_{pq} = - y_{pq}\, e_{pq}$

A set of non-connected elements of a given system is defined as a primitive Network and an element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$$v + e = [z]\, i$$

$$i + j = [y]\, v$$

Primitive network matrices: A diagonal element in the matrices, [z] or [y] is the self impedance $z_{pq\text{-}pq}$ or self admittance, $y_{pq\text{-}pq}$. An off-diagonal element is the mutual impedance, $z_{pq\text{-}rs}$ or mutual admittance, $y_{pq\text{-}rs}$, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Bus Admittance Matrix

Bus admittance matrix (YBus) for an n-bus power system is square matrix of size n × n. The diagonal elements represent the self or short circuit driving point admittances with respect to each bus. The off-diagonal elements are the short circuit transfer admittances (or) the admittances common between any two number of buses. In other words, the diagonal element yii of the YBus is the total admittance value with respect to the ith bus and yik is the value of the admittance that is present between ith and kth buses. YBus can be obtained by the following methods:

1. Direct inspection method

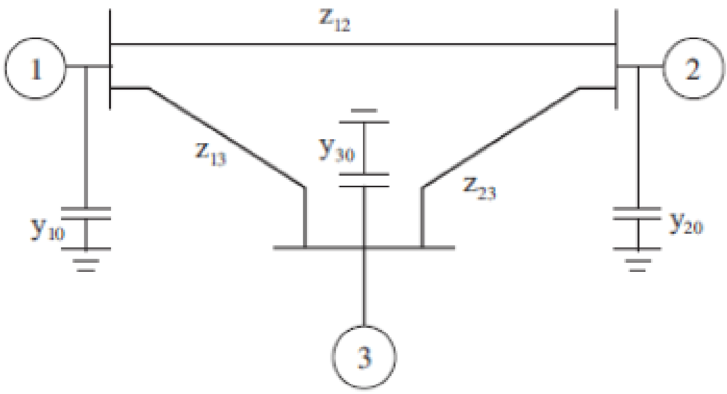
2. Step-by-step procedure

3. Singular transformation

4. Non-singular transformation

Direct inspection method

Formulation of YBus by direct inspection method is suitable for the small size networks. In this method the YBus matrix is developed simply by inspecting structure of the network without developing any kind of equations. Let us consider a 3-bus power system shown in figure below:



Since $[B]^T * i$ is zero because, algebraic sum of all the currents meeting at a node is zero. The source current matrix [j] can be partitioned into,

$$[j] = \begin{bmatrix} j_b \\ j_\ell \end{bmatrix}$$

Where j_b is the source acting in parallel across the branches

$$\therefore [B]^T * [j] = [B]^T * [j_b] = I_{BR}$$

$$I_{BR} = [B]^T * y * [B] * V_{BR}$$

$$I_{BR} = [Y_{BR}] * V_{BR} \quad \text{(or)} \quad I_{BR} = Y_{BR} * V_{BR}$$

$$Y_{BR} = B^T * [y] * B \text{, and}$$

$$Z_{BR} = Y_{BR}^{-1} = \{ B^T * [y] * B \}^{-1}$$

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by n independent nodal equations, where n is the total number of buses (n+1) nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS}\, E_{BUS}$$

where

E_{BUS} = vector of bus voltages measured with respect to reference bus

I_{BUS} = Vector of currents injected into the bus

Y_{BUS} = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y]\, v$$

Pre-multiplying by A^t (transpose of A), we obtain

$$A^t i + A^t j = A^t [y]\, v$$

However, $A^t i = 0$

Since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, $A^t j$ gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence, $A^t j = I_{BUS}$

Thus we have, $I_{BUS} = A^t [y]\, v$

$$v = A\, E_{BUS}$$

$$I_{BUS} = A^t [y]\, A\, E_{BUS}$$

$$Y_{BUS} = A^t [y]\, A$$

The bus incidence matrix is rectangular and hence singular. Hence, the above equation gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1}$$

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances. Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix. The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by (n-1) nodal equations, where n is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.