UNIT II

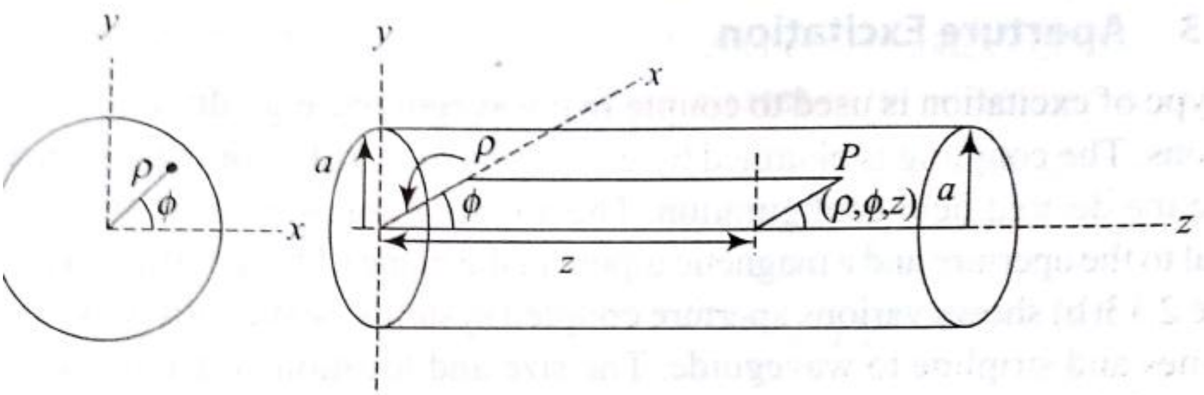
CIRCULAR WAVEGUIDES- MICROSTRIP LINES- CAVITY RESONATORS

CIRCULAR WAVEGUIDES:

INTRODUCTION:

Circular waveguides are basically tubular circular conductors as shown in Fig 1.

A hollow metallic tube of uniform circular cross section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called circular waveguide. Analysis of circular wave guide requires solution of the wave equation in cylindrical coordinates (ρ, φ, z). The direction of propagation is in Z-direction. Maxwell's equations are also expressed in cylindrical coordinates. The electric and magnetic field components along ρ and φ i.e., $H\rho, H\varphi, E\rho$ and $E\varphi$ are expressed in terms of the longitudinal components E_Z and H_Z. The relations are as follows:



$h^2 H\rho = \quad \frac{j\omega\varepsilon}{\rho}\frac{\partial \mathrm{Ez}}{\partial \varphi} \; - \gamma \; \frac{\partial \mathrm{Hz}}{\partial \rho}$

$h^2 H\varphi = \quad - j\omega\varepsilon \frac{\partial \mathrm{Ez}}{\partial \rho} \; - \; \frac{\gamma}{\rho}\frac{\partial Hz}{\partial \varphi}$

$h^2 E\rho = \quad - \gamma \; \frac{\partial \mathrm{Ez}}{\partial \rho} \; - \; \frac{j\omega\mu}{\rho}\frac{\partial \mathrm{Hz}}{\partial \varphi}$

$h^2 E\varphi = \quad - \frac{\gamma}{\rho}\frac{\partial Ez}{\partial \varphi} \; + \; j\omega\mu \frac{\partial \mathrm{Hz}}{\partial \rho}$ Set 1 Equations

where $h^2 = \gamma^2 + \omega^2\mu\varepsilon$ and EQ 2

The wave equations for Ez and Hz in cylindrical coordinates are given by

$$\frac{\partial^2 \mathrm{Ez}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \mathrm{Ez}}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial \mathrm{Ez}}{\partial \rho} + \frac{\partial^2 \mathrm{Ez}}{\partial z^2} + \omega^2\mu\varepsilon \; E_z = 0 \quad \ldots. \; 3 \quad \text{and}$$

$$\frac{\partial^2 \mathrm{Hz}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \mathrm{Hz}}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial \mathrm{Hz}}{\partial \rho} + \frac{\partial^2 \mathrm{Hz}}{\partial z^2} + \omega^2\mu\varepsilon \; H_z = 0 \; \ldots. \; 4$$

Transverse Electric waves

Consider the transverse electric waves Ez =0 So EQ 4 is to be considered.

The boundary condition is the tangential components of electric fields on the cylindrical wall are zero.

We know $\frac{\partial}{\partial z}$ is an operator and is equal to . Then EQ 4 becomes

$$\frac{\partial^2 \mathrm{Hz}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \mathrm{Hz}}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial \mathrm{Hz}}{\partial \rho} + (\gamma^2 + {}^2\mu\varepsilon) \; H_z = 0$$

$$\frac{\partial^2 \mathrm{Hz}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \mathrm{Hz}}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial \mathrm{Hz}}{\partial \rho} + h^2 \, Hz = 0 \; , \quad \ldots..5$$

where $h^2 = (\gamma^2 + \omega^2\mu\varepsilon)$

This is a partial differential equation, whose solution can be obtained by separation of variables method for which it is assumed

Hz = P.Q 6

where P is a function of ρ alone and Q is a function of φ alone.

EQ 5 becomes when EQ 6 is substituted,

$$\frac{\partial^2(PQ)}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2(PQ)}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial(PQ)}{\partial \rho} + h^2 (PQ) = 0$$

On differentiation,

$$Q.\frac{d^2P}{d\rho^2} + \frac{P}{\rho^2}\frac{d^2Q}{d\varphi^2} + \frac{Q}{\rho}\frac{dP}{d\rho} + h^2 (PQ) = 0$$

Multiplying throughout with $\frac{\rho^2}{PQ}$, we get

$$\frac{\rho^2}{P}.\frac{d^2P}{d\rho^2} + \frac{1}{Q}\frac{d^2Q}{d\varphi^2} + \frac{\rho}{P}\frac{dP}{d\rho} + h^2 \rho^2 = 0$$

This can be rearranged as

$$\frac{\rho^2}{P}.\frac{d^2P}{d\rho^2} + \frac{\rho}{P}\frac{dP}{d\rho} + h^2 \rho^2 + \left(\frac{1}{Q}\frac{d^2Q}{d\varphi^2}\right) = 0 \quad .... \; 7$$

Let $\frac{1}{Q}\frac{d^2Q}{d\varphi^2} = -n^2$, --------8 where n^2 is a constant.

Substituting 8 in 7, we get

$$\frac{\rho^2}{P}.\frac{d^2P}{d\rho^2} + \frac{\rho}{P}\frac{dP}{d\rho} + (h^2 \rho^2 - n^2) = 0$$

Multiplying throughout with P,

$$\rho^2.\frac{d^2P}{d\rho^2} + \rho\frac{dP}{d\rho} + P\,(h^2 \rho^2 - n^2) = 0 \quad .....9$$

EQ 9 can be rewritten as

$$(\rho h)^2.\frac{d^2P}{d(\rho h)^2} + (\rho h)\frac{dP}{d(\rho h)} + P\,[(\rho h)^2 - n^2] = 0 \quad ...10$$

This is similar to the Bessel equation of the form

$$x^2 . \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (^2 - n^2)y = 0$$

whose solution is

$y = C_n J_n (x)$, where $J_n (x)$ represents the n^{th} order Bessel function of first kind and C_n is a constant.

Therefore the solution of equation 10 is

$P = C_n J_n (\rho h)$ 9

Also, the general solution of EQ 8 is

$Q = A_n \text{Sin } n\varphi + B_n . \text{ Cos } n\varphi$ 10

Substituting EQ 9 and EQ 10 in EQ 6,

$Hz = C_n J_n (\rho h) (A_n \text{ Sin } n\varphi + B_n \text{ Cos } n\varphi)$ 11

The constants A and B control the amplitudes of sin n φ and cos $n\varphi$ terms which are independent.

Because of the azimuthal symmetry of circular waveguide, both sine and cosine terms are valid solutions. The actual amplitudes of these terms are dependent on the excitation of the waveguide.

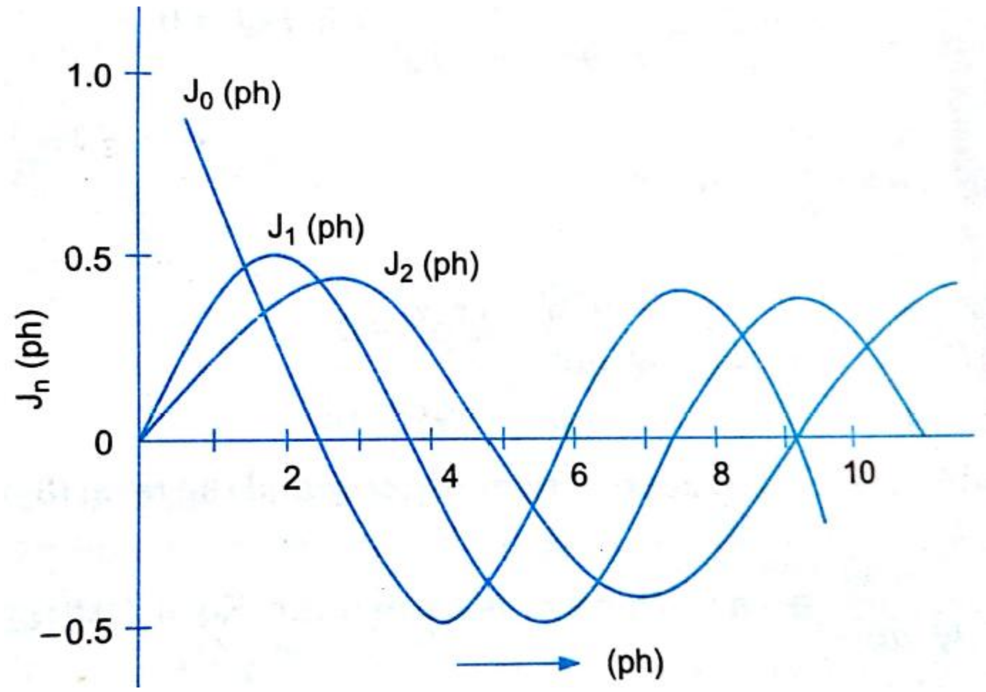
From a different view point, the coordinate system can be rotated about the Z axis to obtain Hz with either A=0 or B=0.

Then we can consider the sinusoidal variation along Z direction with EQ 11 taking the form of

$Hz = C_0 J_n (\rho h) \text{ Cos } n\varphi' \; e^{-\gamma z}$ 12

(Adding the variation along Z direction as $e^{-\gamma z}$)

The n^{th} order Bessel function $J_n(\rho h)$ of the first kind are plotted in Fig below.



Boundarydary condition:

All along the surface of the circular waveguide at ρ =a, E φ = 0 for all values of φ varying between 0 to 2π.

$\frac{\partial H}{\partial \rho}$ =0 at ρ =a This implies $J'_n(ah) = 0$ 13

The prime denotes differentiation with respect to ah. The roots of the equation are defined by P'nm so that $J'_n(P'_{nm})=0$,where the m^{th} root of this equation is denoted by P'_{nm} which are the eigen values given by

$P'_{n,m}$ =ah 14

Or $h = P'_{n,m}/a$ .....15 ,(the permissible values of h is given by this equation)

The equation 12 reduces to

$Hz = C_0 J'_n(\rho h) \cos n\varphi' \; e^{-\gamma z}$ 16

And this equation represents all possible solutions of Hz for $TE_{n,m}$ wave in a circular waveguide. Since J_n are oscillatory functions, $J'_n(ah)$ are also oscillatory function. Substituting the value of Hz (EQ 16) in Set 1 Equations , we get the field components for TE $_{n,m}$ waves in circular waveguide with h=P'n,m/a as given below: **(Z_z (= Eρ /Hφ or –Eφ/Hρ) the wave impedance in the guide**)

$$E\rho = C_{o\rho}\, J_n\left(\frac{P'_{nm}}{a}\rho\right)\sin n\phi\, e^{-\gamma z}$$

$$E\phi = C_{o\phi}\, J'_n\left(\frac{P'_{nm}}{a}\rho\right)\cos n\phi\, e^{-\gamma z}$$

$$E_z = 0$$

$$H\rho = -\frac{C_{o\phi}}{Z_z}\, J'_n\left(\frac{P'_{nm}}{a}\rho\right)\cos n\phi\, e^{-\gamma z}$$

$$H\phi = \frac{C_{o\phi}}{Z_z}\, J_n\left(\frac{P'_{nm}}{a}\rho\right)\sin n\phi\, e^{-\gamma z}$$

$$H_z = C_o\, J_n\left(\frac{P'_{nm}}{a}\rho\right)\cos n\phi\, e^{-\gamma z}$$

The roots of $J_n'(ah)$ correspond to maximum and minimum of the curves $J'_n(ah)$.

The first subscript 'n' denotes the number of full cycles of field variations in one revolution through 2π radians of φ.

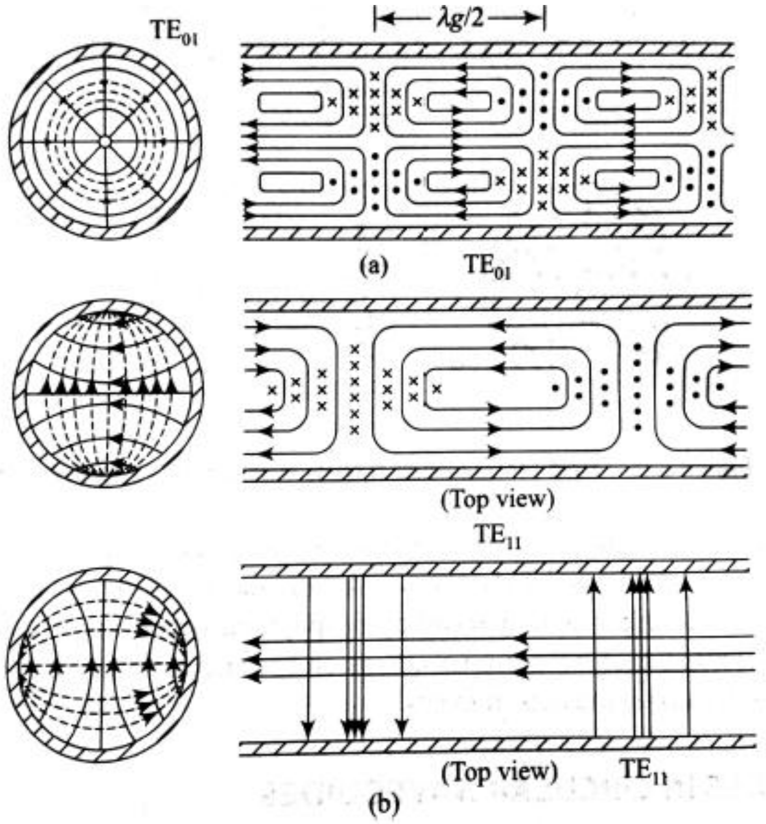
The second subscript 'm 'represents the number of zeros of Eφ, i.e., $J_n'(ah)$ along the radial of waveguide with the exclusion of zero on the axis if it exists.

The values of P’n,m for TE $_{n,m}$ mode (n^{th} order and m^{th} root) in circular waveguide are given in the **table below**.

n \ m	1	2	3
0	3.832	7.016	10.173
1	1.841	5.331	8.536
2	3.054	6.706	9.969
3	4.201	8.015	11.346

Table: Values of P’n,m for TE $_{n,m}$ mode in circular waveguide

Field Configurations of $TE_{n,m}$ modes



____________ E Lines

------------ H LInes

+ inward directed Lines

. outward directed Lines

Transverse Magnetic Modes in circular waveguide

The TM modes in circular waveguide are characterised by Hz=0. However, the Z component of Electric field E must exist in order to have energy transmission in the guide. Consequently the Helmholtz equation for Ez in a circular waveguide is given by

$$\frac{\partial^2 Ez}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 Ez}{\partial \varphi^2} + \frac{1}{\rho}\frac{\partial Ez}{\partial \rho} + \frac{\partial^2 Ez}{\partial z^2} + \omega^2\mu\varepsilon \quad E_z = 0 \quad .... \ 3$$

The solution for the above equation can be obtained on similar lines as in the case of TE wave and the solution comes as

$E_z = C_0 J_n (\rho h) \cos n\varphi' \, e^{-\gamma z}$ 17

The boundary condition is that Ez =0 at ρ =a

Then, $J_n (a h) = 0$.

As $J_n(a h)$ are oscillatory functions ,there are infinite number of roots of $J_n(a h)$.

The values of these roots for which $J_n (a h) = 0$ are called Eigen values and are denoted by $P_{n,m}$ where $P_{n,m}$ = ah.

Table below gives a few of them for lower order n

n \ m	1	2	3
0	2.405	5.520	8.645
1	3.832	7.106	10.173
2	5.135	8.417	11.620
3	6.380	9.761	13.015

Table : Values of $P_{n,m}$ for TM $_{n,m}$ mode in circular waveguide

Substituting Ez (EQ 17) in the set 1 equations, the field components for the $TM_{n,m}$ modes can be written as:

Z_z is, the wave impedance as defined earlier. $E\varphi/H\rho$ or - $E\rho/H\varphi$

$$E\rho = C_{o\rho} \left(\frac{P_{nm}}{a} \cdot \rho \right) \cos n\phi \, e^{-\gamma z}$$

$$E\phi = C_{o\phi} \, J_n \left(\frac{P_{nm}}{a} \cdot \rho \right) \sin n\phi \, e^{-\gamma z}$$

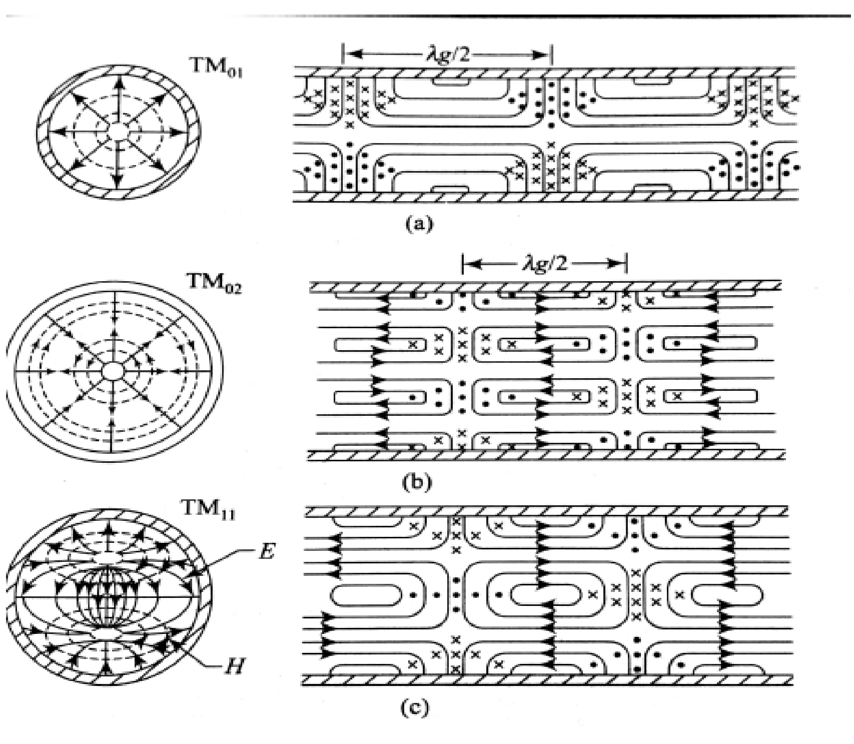
$$E_z = C_{oz} \, J_n \left(\frac{P_{nm}}{a} \cdot \rho \right) \cos n\phi \, e^{-\gamma z}$$

$$H\rho = \frac{C_{o\phi}}{Z_z} \, J_n \left(\frac{P_{nm}}{a} \cdot \rho \right) \sin n\phi \, e^{-\gamma z}$$

$$H\phi = \frac{C_{o\rho}}{Z_z} \, J_n' \left(\frac{P_{nm}}{a} \cdot \rho \right) \cos n\phi \, e^{-\gamma z}$$

$$H = 0$$

The field patterns of TM n,m modes are shown in below Fig.



(Here n=0,1,2,3 and m=1,2,3,4)

Nature of Fields:

______ E Lines

- - - - - H Lines

+ inward directed

. outward directed Lines

The first mode subscript n indicates the number of full wave variations in the circumferential direction, while the second subscript relates to the Bessel function variations in radial direction.

The TE_{11} mode in circular waveguide has similar field patterns as those of TE_{10} in square waveguide. In a gradual change of the guide cross section from square to circular, the TE_{10} mode in the square waveguide becomes TE_{11} mode in circular waveguide

The TM_{01} mode in circular waveguide is analogous to the TM_{11} mode in the square waveguide.

Modes with circular symmetry (TM_{01} and TE_{01}) are utilized in the design of rotary joints.

When rectangular waveguide is used, the plane of polarisation of the propagating wave is uniquely defined. The electric field is directed across the narrow dimension of the waveguide.

When a dual polarisation capability is required especially when a waveguide is connected to a circularly polarised antenna, the waveguide must be able to propagate both the vertically and horizontally polarised waves. A square waveguide has this capacity because a=b and cut off frequencies of TE_{10} and TE_{01} modes are the same.

The circular waveguide is the most common form of a dual polarisation transmission line. Further, they are used in rotational coupling. For the same reason of its circular symmetry, the circular waveguide possesses no characteristic that prevents positively the plane of polarisation of the wave from rotating about the guide axis as the wave travels.

Characteristic Equation and Cut Off Wavelength

$h^2 = (\gamma^2 + \omega^2\mu\varepsilon)$ and $\gamma = \sqrt{h^2 - \omega^2\mu\varepsilon}$

$\gamma = \alpha + j\beta$

i.e.,

$\alpha + j\beta = \sqrt{h^2 - \omega^2\mu\varepsilon} = \sqrt{\mathrm{h}^2\mathrm{n,m} - \omega^2\mu\varepsilon}$

For propagation to start, $\omega^2{}_c\,\mu\varepsilon = h^2{}_{n,m}$ so that,

$f_c = h_{n,m}/2\pi\sqrt{\mu\varepsilon}$

or $\lambda_c = 2\pi/ h_{n,m}$

For TE waves,

$h_{nm} = P'_{nm}/a$ and $\lambda_c = 2\pi a/P'_{n,m}$

The minimum value of P'n,m is 1.841 for n=1 and m=1 for TE waves

and for minimum value of P', the cut off wavelength will be maximum..

For TM waves,

$h_{nm} = P_{nm}/a$. The minimum value of $P_{n,m}$ is 2.405 for n=0 &m=1.

So TM01 mode has the maximum cut off wavelength in TM waves.

So, TE_{11} is the DOMINANT MODE in circular waveguides.

From the Tables, it can be seen that **$P'_{0,m} = P_{1,m}$**

Then, $TE_{0,m}$ and $TM_{1,m}$ modes are **DEGENERATE MODES**.

Phase velocity, Group Velocity, Guide wavelength and Wave Impedance

The relations for phase velocity, group velocity and guide wave length remain the same as in the case of rectangular waveguide for both TE and TM modes.

$$\lambda g = \frac{\lambda}{\sqrt{1-(\frac{\lambda}{\lambda c})^2}}$$

$$\upsilon_p = \frac{c}{\sqrt{1-(\frac{\lambda}{\lambda c})^2}}$$

$$\upsilon_g = c.\sqrt{1-(\frac{\lambda}{\lambda c})^2} \qquad = c.\sqrt{1-(\frac{fc}{f})^2}$$

$\upsilon_p = \omega/\beta$ or $\beta = \omega/\upsilon_p = 2\pi/\lambda g = (2\pi/\lambda) \cdot \sqrt{1-\left(\frac{fc}{f}\right)^2}$

$Z_{TM} = \frac{\beta}{\omega\epsilon} = \sqrt{\mu\varepsilon}\sqrt{(\omega^2 - \omega_c^2)}\ /\omega\varepsilon = \sqrt{\mu/\varepsilon}\ .\sqrt{1-(\frac{\omega c}{\omega})^2}$

$= \sqrt{\mu/\varepsilon}\ \sqrt{1-(\frac{fc}{f})^2} = \eta\ \sqrt{1-(\frac{\lambda}{\lambda c})^2} = \eta\ \lambda\ /\ \lambda g$.

$Z_{TE} = \eta/\sqrt{1-(\frac{\lambda}{\lambda c})^2} = \eta(\lambda g / \lambda)$

Attenuation in Circular Waveguide

The attenuation in circular waveguide for TE and TM modes can be determined with the following definition in the case of circular waveguide also.

The attenuation is defined as

$$\alpha = \frac{\text{Power loss/unit length}}{2\,(\text{Average power transmitted})}$$

Average power transmitted is expressed as

$$(P_{nm})_{av} = \frac{1}{2Z_z}\int_0^{2\pi}\int_0^{a}[\,|E_\phi|^2 + |E_\rho|^2]\,\rho d\,\rho d\phi$$

$$= \frac{Z_z}{2}\int_0^{2\pi}\int_0^{a}[\,|H_\phi|^2 + |H_\rho|^2]\,\rho d\,\rho d\phi$$

For TE_{rm} mode,

$$(P_{nm})_{TE} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta}\int_0^{2\pi}\int_0^{a}[\,|E_\rho|^2 + |E_\phi|^2]\,\rho d\,\rho d\phi$$

and for TM_{nm} modes,

$$(P_{nm})_{TM} = \frac{1}{2\eta\sqrt{1-(f_c/f)^2}} \int_0^{2\pi}\int_0^{a} [\,|E_\rho|^2 + |E_\phi|^2]\,\rho d\rho d\phi$$

The power loss/unit length (over the guide walls)

$$P_L = \frac{R_s}{2} \oint J_s \cdot J_s^* dl$$

The attenuation constant α for TE and TM modes can finally be shown to be

$$\alpha_{TE} = \frac{R_s}{a z_0 \sqrt{1-(f_c/f)^2}} \left[\left(\frac{f_c}{f}\right)^2 + \frac{n^2}{(P'_{nm})^2 - n^2} \right]$$

and
$$\alpha_{TM} = \frac{R_s}{a z_0 \sqrt{1-(f_c/f)^2}}$$

For TM_{0m} modes, attenuation falls off as $f^{3/2}$ as per

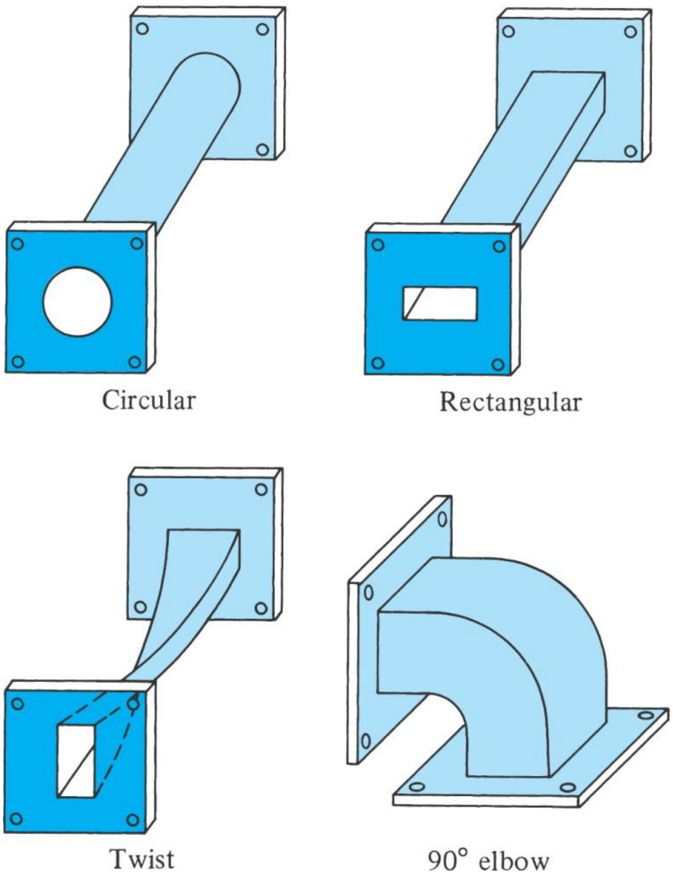
$$\alpha = \frac{R}{a z_0} \frac{f_c^2}{f(f^2 - f_c^2)^{-1/2}}$$

The rapid decrease of attenuation with frequency of TE_{01} mode is useful for long low loss waveguide communication links. But, modes above dominant mode TE_{11} result in mode conversion leading to signal distortion.

Salient Features of Circular waveguides:

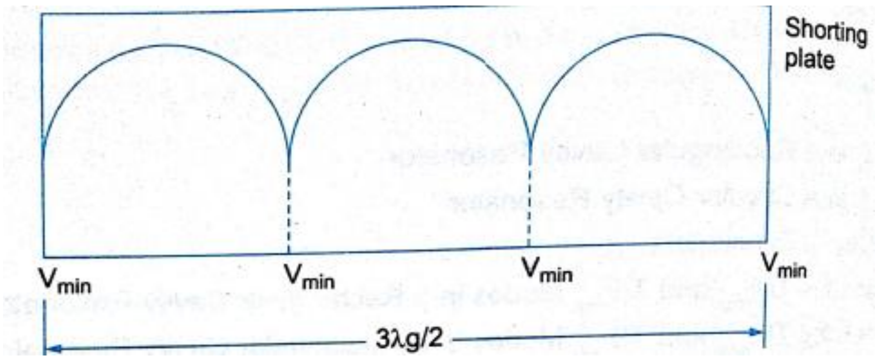
- It is easy to manufacture.
- They are used in rotational coupling.
- Rotation of Polarisation exists and this can be overcome by rotating modes symmetrically.
- TM_{01} mode is preferred to TE $_{01}$ mode as it requires a smaller diameter for the same cut off wavelength.
- For f> 10 GHz, TE_{01} has the lowest attenuation per unit length of the waveguide.
- TE_{01} has no practical application
- The main disadvantage is that its cross-section is larger than that of a rectangular waveguide for carrying the same signal.

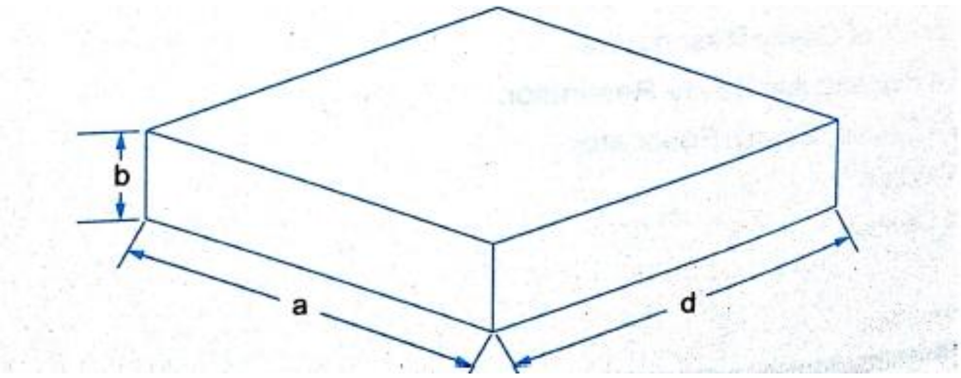
- The space occupied by circular waveguide is more than that of a rectangular waveguide.
- The determination of fields consists of differential equations of certain type, whose solutions involve Bessel Functions.
- It has the advantage of greater power handling capacity and lower attenuation for a given cut off wavelength.



CAVITY RESONATORS

When one end of the waveguide is terminated with a shorting plate, there will be reflections causing standing waves. When another shorting plate is kept at a distance of multiples of λg /2, then the hollow space so formed can support a signal that bounces back and forth between the two shorting plates. This results in resonance. The hollow space is called cavity and the arrangement so done is called **cavity resonator**.





In general, a cavity resonator is a metallic enclosure that confines the electromagnetic energy. The stored electric and magnetic energies inside the cavity determine its equivalent inductance and capacitance. The energy dissipated due to the finite conductivity of the cavity walls determines the equivalent resistance. The metallic enclosure may be a circular or rectangular waveguide sections with shorting plate closing at both ends (Fig above).

A resonator can have an infinite number of resonant modes theoretically, d each mode corresponding to a definite resonant frequency. When the frequency of an impressed signal is equal to a resonant frequency, maximum amplitude of standing wave occurs and the peak energies stored in electric and magnetic fields are equal. The mode having the lowest resonant frequency is known as the **DOMINANT MODE**

Expression for Resonant Frequency

RECTANGULAR WAVEGUIDE,

$h^2 = (\gamma^2 + \omega^2\mu\varepsilon) = (m\pi/a)^2 + (n\pi/b)^2$ 1

$\omega^2\mu\varepsilon = (m\pi/a)^2 + (n\pi/b)^2 - \gamma^2$ 2

For wave propagation to occur $\gamma = j\beta$ 3

Using 3 in 2 we get,

$\omega^2\mu\varepsilon = (m\pi/a)^2 + (n\pi/b)^2 + \beta^2$ 4

For a wave to exist in a cavity resonator, there must be a phase change corresponding to a given guide wavelength.

$\beta\lambda g = 2\pi$ or

$\beta = \pi/(\lambda g/2)$. 5

The distance between the shorting plates, say, d should be multiples of $\lambda g/2$ in order to form the standing waves. . i.e.,

d= p. $\lambda g /2$ ....6.

Substituting the value of $\lambda g /2$ as d/p(from Eq 6) in Eq 5,

β =p π /d .....7.

where, p is an integer

This is the condition for resonance and the resonant frequency ω_o is given by the Eq 4, after substituting ω_o for ω and β =p π /d as

$\omega_o{}^2\mu\varepsilon = (m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2$ ....8.

Or $f_o = \frac{c}{2}\sqrt{(m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2}$ ....9.

General mode of propagation in a cavity resonator is $TE_{m,n,p}$ or $TM_{m,n,p}$. For both TE and TM modes the resonant frequency is the same in rectangular waveguide cavity resonators.

CIRCULAR CAVITY RESONATOR

To short both ends circular end plates are used. Let '**a**' be the radius of the circular waveguide and'd' be the length of the waveguide. The condition for resonance is β =p π /d, as detailed above.

For circular waveguide section,

$h^2 = (\gamma^2 + \omega^2\mu\varepsilon)$

or $h^2 - \gamma^2 = \omega^2\mu\varepsilon$

or $h^2 + \beta^2 = \omega_o{}^2\mu\varepsilon$, applying condition for resonance

$h^2 + (p\pi/d)^2 = \omega_o^2\mu\varepsilon$ For TM $_{nm}$ waves, $h_{nm} = P_{nm}/a$.

. For TE $_{nm}$ waves, $h_{nm} = P'_{nm}/a$

$$f_o = \frac{c}{2\pi}\sqrt{h^2 nm + (p\pi/d)^2} \qquad ......10$$

For TM $_{nm}$ waves, $h_{nm} = P_{nm}/a$

$$f_o = \frac{c}{2\pi}\sqrt{(P\,nm/a)^2 + (p\pi/d)^2} \qquad ......11$$

For TE $_{nm}$ waves, $h_{nm} = P'_{nm}/a$

$$f_o = \frac{c}{2\pi}\sqrt{(P'\,nm/a)^2 + (p\pi/d)^2} \qquad ....12$$

FIELD EXPRESSION for TM modes in Cavity Resonators:

Rectangular cavity Resonator

TM mode

The field expression for TM (Hz =0) wave is

Ez = K Sin [($m\pi/a$) x .Sin (nπ/b) y] $e^{j\omega t - \gamma z}$, ...1.

when the wave is propagating along + direction.

For the return wave i.e., for the wave propagating in – Z direction, EQ 1 becomes

Ez = K Sin [($m\pi/a$) x .Sin (nπ/b) y] $e^{j\omega t + \gamma z}$ ....2.

As the waves propagate γ may be replaced by j β. Adding the fields of the two waves, Ez = K Sin [($m\pi/a$) x .Sin (nπ/b) y] $e^{j\,(\omega t\,\pm\beta z)}$ 3

Let A^+ and A^- be the amplitude constants of onward and backward waves respectively.

Then Ez = ($A^+ \cdot e^{-j\beta z}$ + A^- $e^{+j\beta z}$) K Sin [($m\pi/a$) x .Sin (nπ/b) y] .... 4.

Boundary condition is Ez =0 at z = 0 and at z=d. This can happen only when A^+ and A^- are equal (= A,say). Then EQ 4 becomes

Ez = A ($e^{-j\beta z}$ + $e^{+j\beta z}$) K Sin [($m\pi/a$) x .Sin (nπ/b) y] $e^{j\omega t}$

=2A cos βz K Sin [($m\pi/a$) x .Sin (nπ/b) y] $e^{j\omega t}$ 5

Ez =2KA Cos βz Sin ($m\pi/a$) x .Sin (nπ/b) y $e^{j\omega t}$ 6

At x=0,x=a,y=0 and y=b, Ez =o also the tangential component of Ez i.e., $\frac{\partial \mathrm{Ez}}{\partial z}$ = 0 at z=0 and z=d.

Differentiating EQ 6 w r t z,

0= C sin βd Sin ($m\pi/a$) x .Sin (nπ/b) y $e^{j\omega t}$ at z=d

To make sin βd =0 , βd = p π or β = p π /d. 7

Substituting EQ 7 in EQ 6,

Ez_{mnp} = C Sin ($m\pi/a$) x .Sin (nπ/b) y Cos $(\mathbf{p}\ \boldsymbol{\pi}\ /\mathbf{d})z$. $e^{j\omega t-\gamma z}$,

where m=o,1,2, 3 .... Represents number of half cycles in x direction,

n= 0,1,2,3...... Represents number of half cycles in y direction,

and p= 0,1,2,3...... Represents number of half cycles in z direction.

TE Mode

For TE wave Ez =0 and Hz = K $\left\{\text{Cos}\ \frac{m\pi}{a}\text{x}\,.\,\text{Cos}\frac{n\pi}{b}\text{y}\ \right\} e^{j\omega t\ -\gamma z}$ 1

For the waves travelling both ways, with $\gamma = j\beta$, the equation changes to

Hz = K $\left\{\text{Cos}\ \frac{m\pi}{a}\text{x}\,.\,\text{Cos}\frac{n\pi}{b}\text{y}\ \right\} e^{j\omega t\ \pm\beta z}$

=($A^{+}\ e^{-j\beta z}$ + $A^{-}\ e^{+j\beta z}$) K $\left\{\text{Cos}\ \frac{m\pi}{a}\text{x}\,.\,\text{Cos}\frac{n\pi}{b}\text{y}\ \right\}\ e^{j\omega t}$ 2

We know, E_y= $-\frac{\gamma}{h^2}\frac{\partial Ez}{\partial y} + \frac{\text{j}\omega\mu}{\text{h}^2}\frac{\partial Hz}{\partial x} = \frac{\text{j}\omega\mu}{\text{h}^2}\frac{\partial Hz}{\partial x}$ since Ez =0 ...3

$E_y = \frac{\text{j}\omega\mu}{\text{h}^2}$ K[($A^{+}\ e^{-j\beta z}$ + $A^{-}\ e^{+j\beta z}$)(-mπ/a)($\sin\ \frac{m\pi}{a}\text{x}\,.\,\text{Cos}\frac{n\pi}{b}\text{y})\ e^{j\omega t}$ 4

Since Ey =0 at z=0 and z=d,

($A^{+}\ e^{-j\beta z}$ + $A^{-}\ e^{+j\beta z}$) =0 5

We choose $A^{+} = -A^{-}$ (to make Ey =0) 6

It is merely necessary to choose the harmonic functions in Z to satisfy the boundary condition of zero tangential E at the remaining two end walls

Substituting 6 in 5,

$A^{+}\ (e^{-j\beta z}\ -\ e^{+j\beta z}\ ) = 0$

i.e., 2j A^{+} Sin βz =0 at z=d

Sin βd = 0 βd = 0 or pπ or β = pπ /d 5

Then, **Hz = K** $\boldsymbol{\text{Cos}\ \frac{m\pi}{a}\text{x}\,.\,\text{Cos}\frac{n\pi}{b}\text{y}\ \ \sin\frac{p\pi}{d}\text{z}\ \ e^{j\omega t-\gamma z}}$

Circular cavity resonators

TE MODE:

We have Hz = C_0 J_n (ρh) Cos nφ' $e^{j\omega t - \gamma z}$

The combined field equation for propagation to and fro is

Hz = Cn J_n (ρh) Cos nφ' $e^{j(\omega t \pm \beta z)}$

Hz=($A^+ e^{-\beta z}$ + $A^- e^{+\beta z}$) C_0 J_n (ρh) Cos nφ' $e^{j\omega t}$

Since Hz cannot be made equal to zero..

Eφ and Eρ can be made equal to zero and to make Eφ and Eρ vanish at z=0 and z=d We choose $A^+ = A^-$

Then the factor ($A^+ e^{-\beta d}$ - $A^+ e^{+\beta d}$) =-2j A^+ sin βd

For sin βd to become zero βd =pπ. Then β =pπ /d, where p=1,2,3,4....

Then,

Hz = Cn J_n (ρh) Cos nφ' sin (pπ /d)z $e^{j(\omega t - \beta z)}$

where

n=0,1,2,3.......is the number of full cycle variations in azimuthal φ direction

m=1,2,3,4......is the number of full cycle variations in radial ρ direction

p=1,2,3,4.....is the number of half cycle variations in axial Z direction.

TM Mode:

We have $Ez = Cn\ J'_n\ (\rho h)\ \text{Cos}\ n\varphi'\ \ e^{j(\omega t - \beta z)}$

The combined wave form is

$Ez = Cn\ J'_n\ (\rho h)\ \text{Cos}\ n\varphi'\ \ e^{j(\omega t \pm \beta z)}$

$Ez ==(A^+ \cdot e^{-\beta z} + A^- e^{+\beta z})\ Cn\ J'_n\ (\rho h)\ \text{Cos}\ n\varphi'\ e^{j\omega t}$

To make Ez = 0 t z=0 and Z=d, we choose $A^+ = - A^-$

$0 = A^+ (\cdot e^{-\beta z} - e^{+\beta z})\ Cn\ J'_n\ (\rho h)\ \text{Cos}\ n\varphi'\ e^{j\omega t}$

$(\cdot e^{-\beta z} - e^{+\beta z}) = -2j \sin \beta z$

But at Z=0 and z=d, Ez =0

$0 = 2j\ A^+ [Cn\ J'_n\ (\rho h)\ \text{Cos}\ n\varphi'\ e^{j\omega t}]\ \sin\beta d$

This can be zero only when $\sin\beta d = 0$ i.e., $\beta d = p\pi$ or $\beta = \frac{p\pi}{d}$ where p =1,2,3,...

Then, $\mathbf{Ez = Cn\ J'_n\ (\rho h)\ Cos\ n\varphi' Sin(\frac{p\pi}{d} z)\ \ e^{j(\omega t - \gamma z)}}$ To Summarise,

Expressions for resonant frequency of a cavity resonator

Rectangular Cavity Resonator (same for TE_{mnp} and TM_{mnp} Modes

$$\mathbf{f_o = \frac{c}{2}\sqrt{(m\pi/a)^2 + (n\pi/b)^2 + (p\,\pi\,/d)^2}}$$

Circular cavity resonator

$$\mathbf{f_o = \frac{c}{2\pi}\sqrt{(P'\,nm/a)^2 + (p\,\pi\,/d)^2}} \qquad \textbf{For } \mathbf{TE_{nmp}} \textbf{ mode}$$

$$\mathbf{f_o = \frac{c}{2\pi}\sqrt{(P\,nm/a)^2 + (p\,\pi\,/d)^2}} \qquad \textbf{For } \mathbf{TM_{nmp}} \textbf{ mode}$$

Expressions For Field Equations

Rectangular cavity resonator

Hz_{mnp} = K $\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y$ **$\sin\left(\frac{p\pi}{d}\right)z$** $e^{j\omega t-\gamma z}$ **for TE_{mnp} mode**

Ez_{mnp} = K **$\sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y$ $\cos\left(\frac{p\,\pi}{d}\right)z \;.\; e^{j\omega t-\gamma z}$** **for TM_{mnp} mode**

where m=o,1,2, 3 .... Represents number of half cycles in x direction,

n= 0,1,2,3...... Represents number of half cycles in y direction,

and p= 0,1,2,3...... Represents number of half cycles in z direction.

Circular cavity resonator

Hz = Cn J_n (ρh) Cos nφ' sin (pπ /d)z $e^{j(\omega t-\gamma z)}$ **for TE_{nmp} mode**

Ez = Cn J'_n (ρh) Cos nφ' $Sin(\frac{p\pi}{d}z)$ $e^{j(\omega t-\gamma z)}$ **for TM_{nmp} mode**

where

n=0,1,2,3.......is the number of full cycle variations in azimuthal φ direction

m=1,2,3,4......is the number of full cycle variations in radial ρ direction

p=1,2,3,4.....is the number of half cycle variations in axial Z direction.

In the rectangular cavity resonator, dominant mode is TE_{101} mode for a>b<d

In circular cavity resonator, TM110 mode is dominant mode where 2a>d and TE111 mode is dominant mode when d $\geq$ 2a

Q factor and coupling Coefficients:

The quality factor Q is a measure of the frequency selectivity of a resonant or anti resonant circuit. It is defined as

$$Q = 2\pi\frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}} = \frac{\omega W}{P} \qquad ......1$$

where W is the maximum energy stored and P is the energy power loss.[2]

At resonant frequency, the electric and magnetic energies are equal and in quadrature. When the electric energy is maximum the magnetic energy is zero and vice versa. So, the total energy stored in the resonator is obtained by integrating the energy density over the volume of the resonator.

$$W_e = \int \frac{\varepsilon}{2}|E|^2 dv = Wm = \int \frac{\mu}{2}|H|^2 dv = W, \qquad ....2$$

where W_e is the electrical energy ,W_m is the magnetic energy , $|H|$ and $|E|$ are the peak values of magnetic and electrical field intensities.

The average power loss in the resonator can be evaluated by integrating the power density $\frac{1}{2}\int |H|^2 R_s$ over the inner surface of the resonator. ($R_s = \sqrt{\frac{\mu\omega}{2\sigma}}$, is the surface resistance)

$$P = \frac{Rs}{2}\int |Ht|^2 da \qquad ....3$$

where H_t is the peak value of the tangential magnetic field intensity and R_s is the surface resistance of the resonator.

Substituting 2 and 3 in 1,

$$Q = \frac{\omega\int\frac{\mu}{2}|H|^2 dv}{\frac{Rs}{2}\int |Ht|^2 da} = \frac{\mu\omega\int |H|^2 dv}{Rs\int |H_t|^2 da} \qquad ....4$$

Since the peak value of the magnetic intensity is related to its tangential and normal components, $|H|^2 = |H_t|^2 + |H_n|^2$, where H_n is the peak value of the normal magnetic field intensity. The value of $|H_t|^2$ at the resonator walls is approximately equal to twice the value $|H|^2$ averaged over the volume.

So, the Q of a cavity resonator as given by EQ 4 can be expressed approximately by

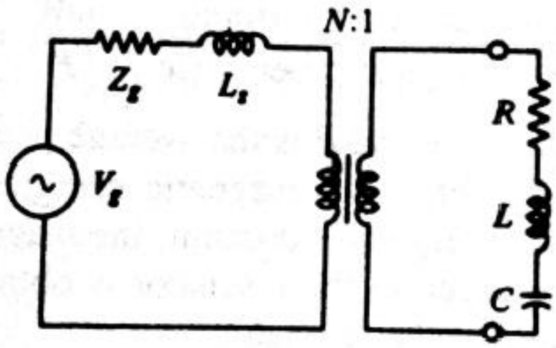
$Q = \omega\mu(\text{Vol})/2R_s(\text{surface area})$ 5

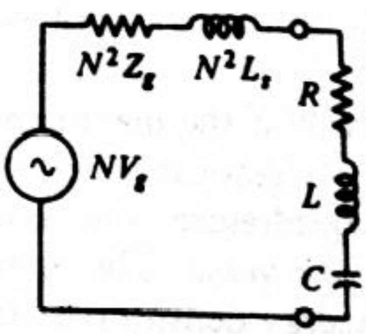
An unloaded resonator can be represented by either a series or a parallel resonant circuit. The resonance frequency f_o and the unloaded Q factor Q_o of a cavity resonator are

$f_o = 1/2\pi\sqrt{LC}$ 6

and $Q_o = L\omega_o/R$ 7

If the cavity is coupled by means of an ideal N:1 transformer and a series inductance Ls to a generator having an internal impedance Zg, then the coupling circuit and its equivalent appear to be as follows.





A. Coupling Circuit b. Equivalent Circuit

The loaded Q_L of the system is given by

$Q_L = L\omega_o/(R+N^2Zg+N^2Ls) = L\omega_o/(R+N^2Zg)$ as $|N^2Ls| << |R+N^2Zg|$

This can be written as $Q_L = L\omega_o/R(1+N^2Zg/R)$ 8

The coupling coefficient of the system is defined as

$K = N^2Zg/R$ 9

And the loaded Q_L would become

$Q_L = L\omega_o/R(1+K) = Q_o/(1+K)$ 10

Rearranging EQ 10,

$1/Q_L = (1/Q_o) + (1/Q_{ext})$ 11

Where $Q_{ext} = Q_o/K = L\omega_o/KR$ is the external Q

There are three types of coupling coefficients.

1. **Critical Coupling:** If the resonator is matched to generator, then K=1....12

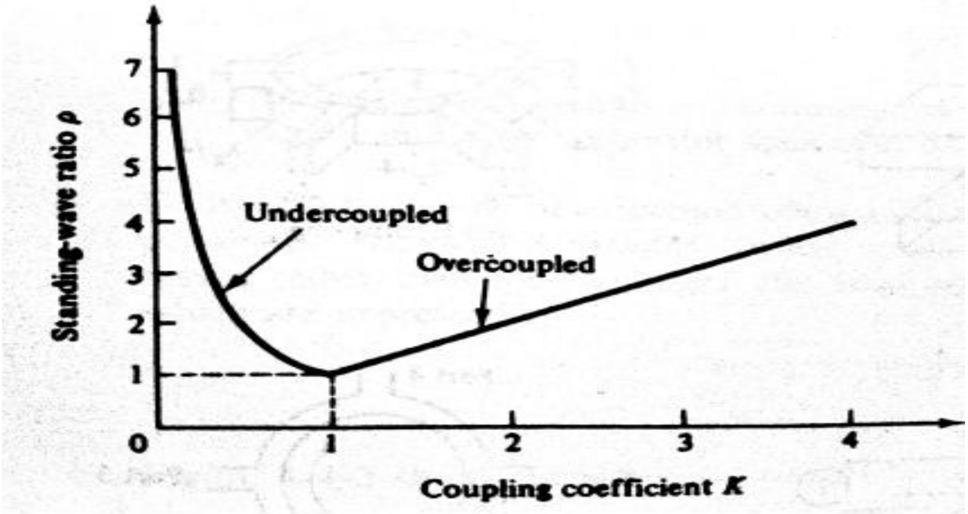
Then, the loaded Q_L is given by (from EQ 10) $Q_L = \frac{1}{2} Q_o$ 13

2. **Over Coupling:** If K>1, the cavity terminals are at voltage maximum in the input line at resonance. The normalised impedance at the voltage maximum is the standing wave ratio ρ. $K = \rho$ 14

The Loaded QL is given by $Q_L = Q_o/(1+\rho)$ 15

3. **Under Coupling:** If K<1, the cavity terminals are at a voltage minimum and the input terminal impedance is equal to the reciprocal of the standing wave ratio ρ i.e., $K = 1/\rho$ ....16 Then $Q_L = \frac{\rho}{\rho+1} Q_o$ 17

The relationship of coupling coefficient and the standing wave ratio is shown in the Fig



The unloaded quality factor Q_o of a cavity resonator can be defined in different ways, such as,

Q_o = (Volume of the cavity) / (skin depth) X(surface area of the cavity). Or,

Q_o = Cross sectional area of the cavity /(Skin depth) X (periphery of the cavity).

Thus the Q factor of a cavity can be increased by increasing the size of the cavity or conductivity of the walls or by decreasing the coupling into the cavity. Q also increases with an increase in frequency as skin depth decreases with frequency.

Q of a circular cavity resonator is given by

Q= $\frac{1}{2}\frac{1}{\mathrm{Rs}}\frac{\beta^2}{\omega}[\frac{ac}{a+c}]$, where ω is the angular frequency, R_s is surface resistance, a is radius and c is the wall length of the circular cavity resonator. These cavity resonators are used widely at frequencies above 3 GHz. The quality of these resonators can be quite high at microwave frequencies with typical values of unloaded Q ranging from 5000 to 50,000.

EXCITATION TECHNIQUES- Waveguides and Cavities:

In general, the field intensities of desired mode in a waveguide can be established by means of a probe or loop coupling devices.

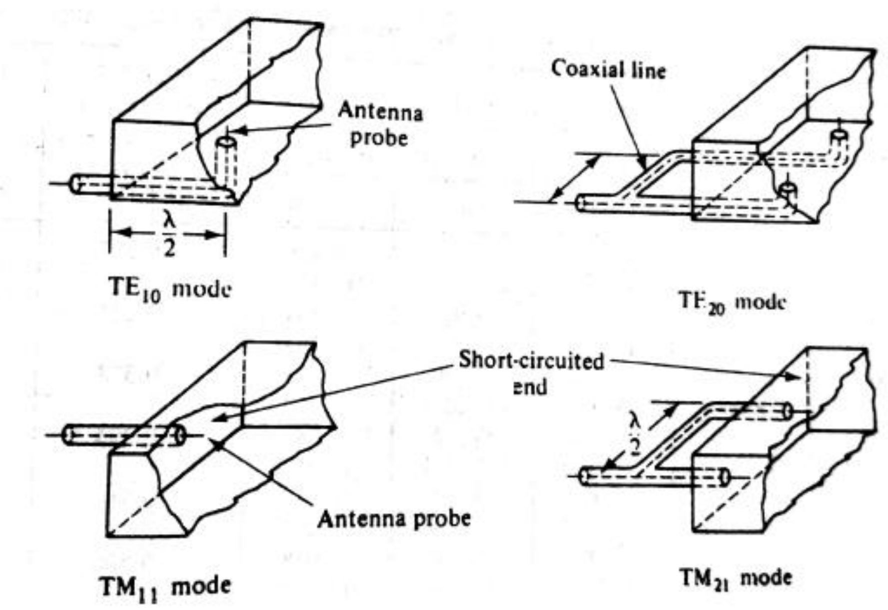
The probe may be called a monopole antenna. The coupling loop may be called a loop antenna.

A probe should be located so as to excite the electric field intensity or a coupling loop should be located so as to generate the magnetic field intensity for a desired mode.

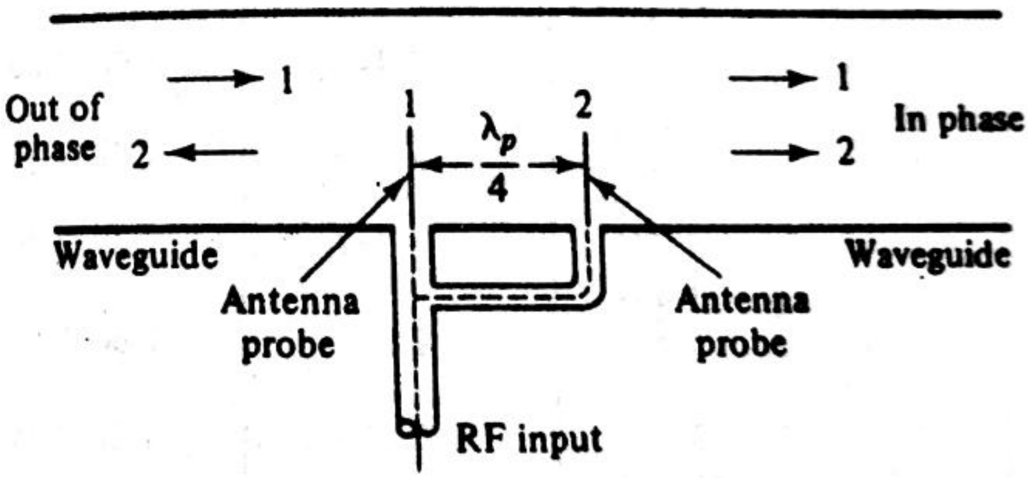
A device that excites a given mode in the guide can also serve reciprocally as a receiver or collector of energy of that mode.

EXCITATION OF MODES IN RECTANGULAR WAVEGUIDES:

The methods of excitation for various modes in rectangular waveguides are shown in Figure.



In order to excite a TE10 mode in one direction of the guide, the two exciting antennas should be arranged in such a way that the field intensities cancel each other in one direction and reinforce in the other. Figure shows a method to launch a TE10 mode in one direction. The two antennas are placed a quarter wave length apart and their phases are in time quadrature. Phasing is compensated by use of additional quarter wavelength section of line connected to the antenna feeders.

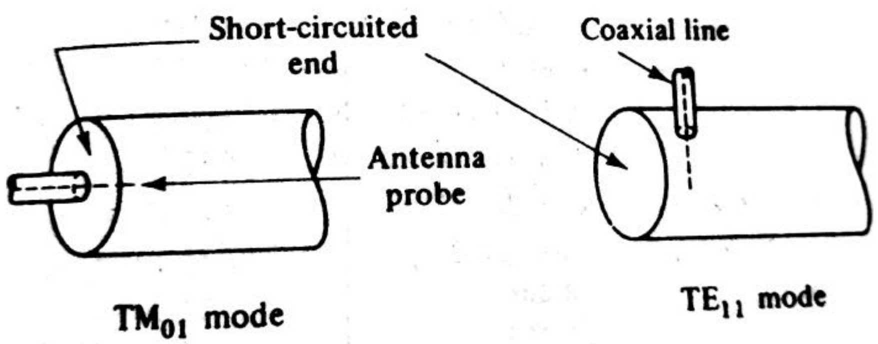


The field intensities radiated by the two antennas are in phase opposition to the left of the antenna and cancel each other, whereas in the region to the right of the antenna, the field intensities are in time phase and reinforce each other. The resulting wave thus propagates to the right in the guide.

Some higher order modes may form due to discontinuities, but they get attenuated. The dominant mode tends to remain the same even when the waveguide is large enough to support the higher modes.

EXCITATION OF MODES IN CIRCULAR WAVEGUIDES:

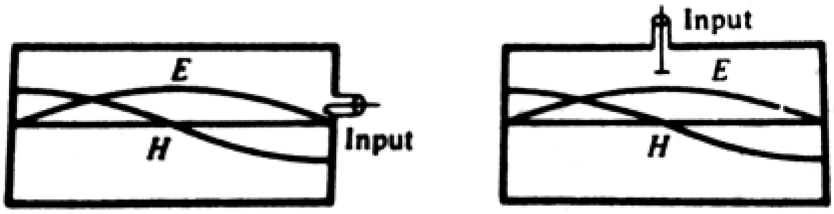
TE modes have no Z component of electric field and TM modes have no Z component of magnetic field intensity. If a device is inserted in a circular waveguide in such a wave as to excite only a Z component of electric field, the wave propagating through the guide will be TM mode: on the other hand, if a devise is placed in a circular waveguide in such a manner so as to excite Z component of magnetic field intensity, the wave propagating will be TE mode. The methods of excitation for various modes in circular waveguides are shown in Figure.



A common way to excite TM modes in circular waveguide is by a coaxial line. At the end of the coaxial wire a large magnetic intensity exists in φ direction of wave propagation. The magnetic field from coaxial line will excite the TM modes in the guide.

EXCITING WAVE MODES IN RESONATOR:

In general a straight –wire probe inserted at the position of maximum electric intensity is used to excite a desired mode, and loop coupling placed at the position f maximum magnetic intensity is utilised to excite a specific mode.



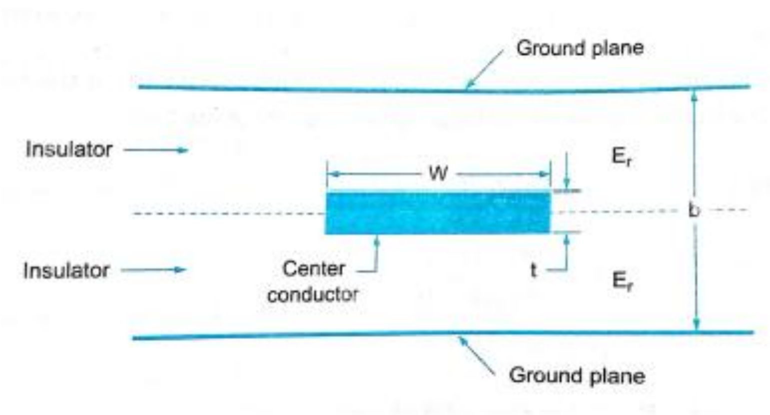
The figure shows the method of excitation for the rectangular resonator. The maximum amplitude of standing wave results when the frequency of the impressed signal is equal to the resonant frequency.

MICRO STRIP LINES:

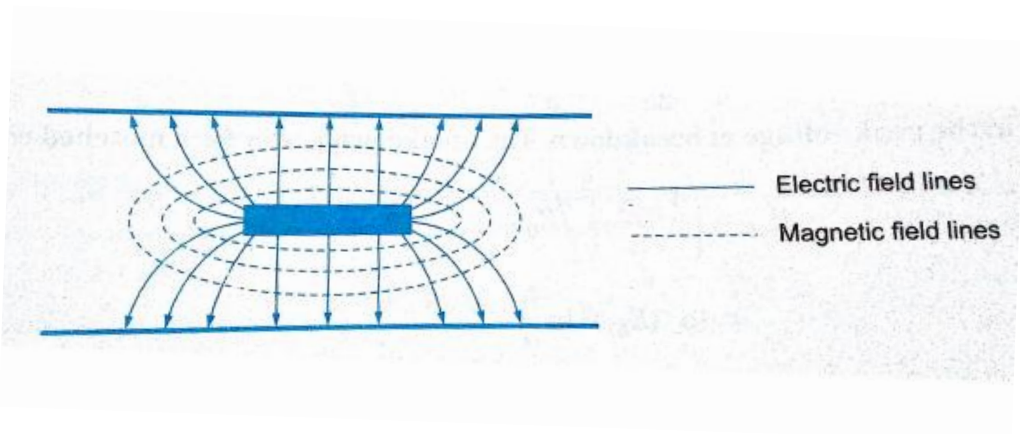
INTRODUCTION:

Strip lines are essentially modifications of to wire lines and coaxial lines. They are basically planar transmission lines widely used at frequencies 100MHz to 100 GHz.

A central conducting thin strip of width w and thickness t is placed inside low loss dielectric substrate(E_r). The substrate is between two metallic ground plates, the width of the ground plates being five times the spacing 'b' between them. (w>t)



The dominant mode is TEM mode. For b< $\lambda/2$, there will be no wave propagation in transverse direction. The field configuration in the strip line is shown in the Fig below.



MICRO STRIP LINES

Before the advent of monolithic microwave integrated circuits(MMIC), parallel striplines and wave guides were much in use. Later the microstrip lines are in extensive use as hey provide one free and accessible surface on which the solid state devices can be placed.

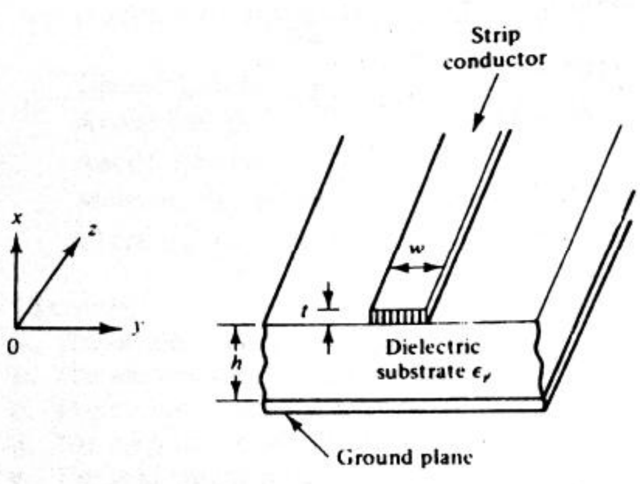
The microstrips are also called open striplines or surface waveguides.

A microstrip line is an unsymmetrical strip line but a parallel plate transmission line having dielectric substrate, one face of which is metallised ground and the other has a thin conducting strip of certain width w and thickness t In this the top ground plate is absent but sometimes a cover plate is used to shield the microstrip line without affecting the field lines.

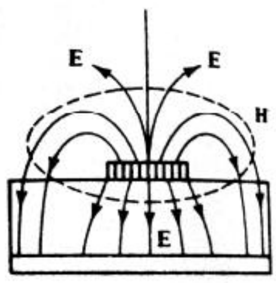
Microstrips are used extensively to interconnect high speed logc circuits in digital computers because they can be fabricated by automated techniques and they provide the required uniform signal paths.

Microstrip line consists of a conduction ribbon attached to a dielectric sheet with conductive backing.

FIELD PATTERN: Modes on the stripline are quasi-TEM modes.



(a) Schematic diagram



(b) Field lines

The theory of TE or TM coupled lines applies as an approximation only. The approximate field distribution is shown in the above Fig in b, where as a is the schematic diagram of the microstrip line.

The distribution of the electric field lines indicates that the E lines approach air-dielectric interface obliquely. And thus there are at least two components of electric field. Since the tangential component of electric field is continuous at the air- dielectric interface, the tangential component of displacement density becomes discontinuous.

$(\nabla \times H)x \;|_{\text{air}} \neq (\nabla \times H)x \;|_{\text{dielectric}}$, ...1

where direction x is tangential to dielectric surface and perpendicular to the strip conductor. For TEM wave Hz =0. Then , EQ 1 gives

$\frac{\partial Hy}{\partial z} \;|_{\text{air}} \neq \frac{\partial Hy}{\partial z} \;|_{\text{dielectric}}$, or

$Hy \;|_{\text{air}} \neq Hy \;|_{\text{dielectric}}$ 2

The inequality in EQ 2 violates the field matching conditions for the normal components of magnetic field.(Y direction is normal to the strip and substrate and the wave propagation is along Z direction)

This implies that Hx should be a non zero quantity for EQ 1 to be satisfied. This leads to the conclusion that a pure TEM wave cannot be supported by a microstrip line.

However, since the major portion of electric field lines is concentrated below the strip, the electric flux crossing the air-dielectric boundary is small. Therefore the deviation from the TEM mode is small and may be ignored for most of the circuit design applications.

Advantages and Disadvantages of Microstrip Lines

1. Substrates of high dielectric constant are advantageous since they reduce the phase velocity and guide wavelength and consequently the circuit dimensions also.

2. Complete conductor pattern may be deposited on a single dielectric substrate which is supported by a single metal ground plane. Fabrication costs would be substantially less than those of coaxial, waveguide or stripline circuits.

3. Because of easy access to the top surface, it is easy to mount any passive or active discrete devices and also for making minor adjustments after fabrication. Access will be there for probing and measurement purposes.

4. Due to planar nature of micro strip structure, both packaged and unpackaged semiconductor chips can be conveniently attached to the micro strip element.

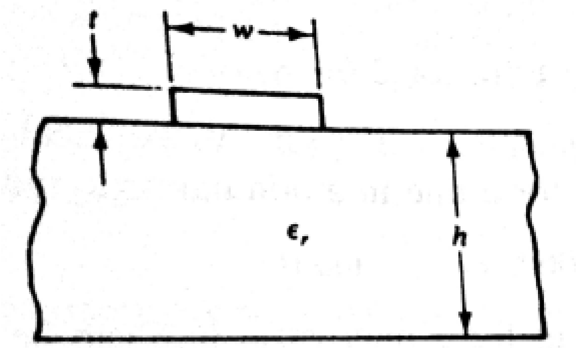
5. Radiation loss in micro strip lines particularly at discontinuities like corners, short circuit posts etc may be reduced considerably by the use of thin and high dielectric materials to ensure the fields confined near the strip.

Because of proximity of the air-dielectric-air interface with the micro strip conductor, at the interface a discontinuity in the electric and magnetic fields is generated. This results in a micro strip configuration that becomes a mixed dielectric transmission structure with impure TEM modes propagating. This makes the analysis complicated.

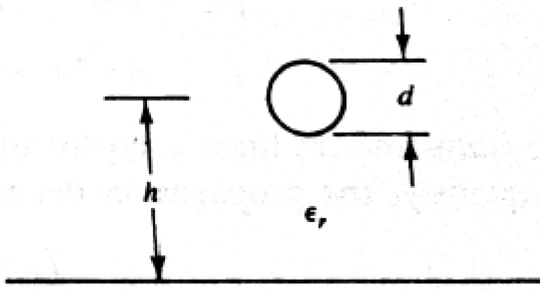
Characteristic Impedance Z_o

The characteristic impedance of a micro strip line i a function of the strip line width, thickness, the distance between the line and the ground and the homogeneous dielectric constant of the board material.

Taking the equation of the characteristic impedance of a wire over ground transmission line, an indirect or comparative method is evolved for Z_o of the micro strip line.



a. Cross-section of Micro strip line



b. Cross- section of wire –over-ground line

The characteristic impedance of a wire –over-ground line is given by

$Z_o = (60/\varepsilon_r)\ \ln \frac{4h}{d}$ for h>>d 1

Where ε_r is the dielectric constant of the ambient medium, h is the height of the centre of wire to the ground and d is the diameter of the wire.

Effective dielectric constant:

For a homogeneous dielectric medium, the propagation delay time Td per unit length is given by

Td =$\sqrt{\mu\varepsilon}$ (μ *and* ε are permeability and permittivity of the medium)

In free space Td =1.016$\sqrt{\varepsilon r}$

The effective relative dielectric constant ε_{re} can be related to the relative dielectric constant of the board material ε_r. The empirical equation is given by

ε_{re} = 0.475 ε_r + 0.67 2

The cross section of micro strip line is rectangular and this rectangular conductor must be transferred into an equivalent circular conductor. The empirical relation of this transformation is given by

D= 0.67 w (0.8 + $\frac{t}{w}$) ....3.

where d is the diameter of the wire over ground, w is the width of the micro strip line and t is the thickness of the micro strip line, provided t/w should lie between 0.1 and 0.8

Substituting 2 for dielectric constant and 3 for equivalent diameter in 1,

Z_o = $\frac{\mathbf{87}}{\sqrt{\varepsilon r+1.41}}$ ln $\frac{5.98h}{0.8w+t}$ ......4 (for h < 0.8w)

EQ 4 is the equation of characteristic impedance for a narrow micro strip line. The characteristic impedance for a wide micro strip line is expressed as

Z_o = $\frac{\mathbf{377}}{\sqrt{\varepsilon r}}\ \frac{\boldsymbol{h}}{\boldsymbol{w}}$ (for w>>h) 5

Losses in Micro strip Lines

The attenuation constant of the dominant mode of the micro strip line depends on geometric factors, electrical properties of substrate and conductors and the frequency.

For non magnetic dielectric substrate, there occur two types of losses, one due to dielectric in the substrate and another due to ohmic skin loss in the strip conductor and the ground plate.

Dielectric Losses

When the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase.

The intrinsic impedance of dielectric is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}\,(1-j\frac{\sigma}{\omega\varepsilon})^{-1/2} \quad ......1$$

And propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = j\omega\sqrt{\mu\varepsilon}\,(1-j\frac{\sigma}{\omega\varepsilon})^{1/2}$ 2

The term $\sigma/\omega\varepsilon$ is referred to as the loss tangent and is defined by

$\text{Tan}\theta = \sigma/\omega\varepsilon$ 3

If $\sigma/\omega\varepsilon << 1$, the propagation constant can be calculated by the binomial expansion as

$$\gamma = j\omega\sqrt{\mu\varepsilon} - j\omega\sqrt{\mu\varepsilon}\,(-j\frac{\sigma}{2\omega\varepsilon}) = j\omega\sqrt{\mu\varepsilon} - \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad ....4$$

From the equation 4, the attenuation part is $\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ which is the dielectric attenuation constant, α_{d}.

$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ Np/cm 5

σ is the conductivity of dielectric substrate

From 3 and 5, eliminatingσ,

$\alpha_d = \frac{\omega}{2}\sqrt{\mu\varepsilon}\ \tan\theta$ Np/cm ......6 or $\alpha_d = 4.34\omega\sqrt{\mu\epsilon}\ \tan\theta$ dB/cm 7

Ohmic Losses:

In micro strip line over a low loss dielectric substrate, the predominant sources of losses at microwave frequencies are the non perfect conductors. The current density in the conductors of a micro strip line is concentrated in a sheet that is approximately a skin depth thick inside the conductor surface and exposed to the electric field. Further, the current density in the strip conductor and ground conductor is not uniform in the transverse plane. This attenuation in the conductor is given by α_c equal to

$\alpha_c = \frac{8.686\ \mathrm{Rs}}{\mathbf{Z_o w}}$ dB/cm for w/h >1, where R_s is the surface resistance given by $\sqrt{\frac{\pi f\mu}{\sigma}}$ = $1/\delta\sigma$ where δ is the skin depth (= $\frac{1}{\sqrt{\pi f\mu\sigma}}$)

Radiation Losses

The radiation loss depends upon the substrate's thickness and dielectric constant as well as its geometry. The radiation factor decreases with increasing substrate dielectric constant. The radiation loss decreases when characteristic impedance increases